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Mathematics Curriculum Standards

Foundations I

Course Description:

Students who enter high school not ready for Algebra I will explore and apply concepts, processes, and skills that are essential to successfully completing the high school graduation requirements in mathematics. Students may receive mathematics credit for only one of the following courses:
Foundations I, Foundations II, and Mathematics for Technology I.

Through the investigation of meaningful problems individually or in cooperative groups, while using appropriate technology, students will strengthen their foundations of mathematics. Students will prepare for success in future mathematics courses by building content knowledge to meet standards in number and operations, algebra, geometry, measurement, and data analysis and probability. The processes of problem solving, reasoning, communication, connections, and representation are interwoven throughout the content standards.

Content Standard 1.0: Number and Operations

Students will develop number and operation sense needed to represent numbers and number relationships verbally, symbolically, and graphically and to compute fluently and make reasonable estimates in problem solving.

Learning Expectations:

The student will:

1.1 demonstrate an understanding of the subsets, elements, properties, and operations of the rational number system;
1.2 connect physical, graphical, verbal, and symbolic representations of rational numbers;
1.3 order and compare rational numbers;
1.4 informally describe and model the concept of additive and multiplicative inverses (e.g., opposites, reciprocals) in real life problem situations;
1.5 apply number theory concepts (e.g., primes, composites, factors, divisibility ,and multiples) in mathematical problem situations;
1.6 use rational numbers to represent real-world applications (e.g., probability, proportionality);
1.7 use mathematical notations appropriately;
1.8 select and apply an appropriate method (i.e., mental arithmetic, paper and pencil, or technology) for computing with rational numbers, and evaluate the reasonableness of results;
1.9 apply estimation strategies in computation and in problem solving.

Student Performance Indicators:

At Level 1, the student is able to

- represent rational numbers in a variety of ways using concrete objects, pictures, the number line, and symbols;
- compute fluently with whole numbers, fractions, decimals, and percent;
- compare rational numbers using the appropriate symbol (<,>,=);

- work flexibly with fractions, decimals, and percents in one-or two-step word problems.

At Level 2, the student is able to

- represent quantitative relationships using ratios and proportions;
- identify numbers less than 100 as prime or composite;
- give the prime factorization for a number less than 100;
- determine the greatest common factor and the least common multiple for up to three numbers;
- apply order of operations in computing with rational numbers, using no more than two parentheses and exponents 1 and 2;
- calculate rates involving cost per unit to determine the best buy;
- demonstrate an understanding of percent in solving real-world problems;
- describe relationships among operations
- use estimation strategies to select a reasonable solution to a real-world problem involving rational number computation.

At Level 3, the student is able to

- develop, analyze, and explain methods for solving problems involving proportions (i.e., scaling, finding equivalent ratios);
- express a monomial written in expanded form using exponents;
- identify the opposite and the reciprocal of a given rational number;
- determine square roots of perfect squares (<169).

Sample Task: Use newspapers to find examples of rational numbers and express them in alternate forms.

Linkages: Make connections to other disciplines by listing examples of whole numbers, fractions, decimals, percents, and integers used in social studies (latitude/longitude), science (scientific notation), business (stock prices, mark down percents), and other disciplines or careers.

Content Standard 2.0: Algebra

Students will describe, extend, analyze, and create a wide variety of patterns and solve real-world problems using appropriate representations.

Learning Expectations:

The student will:

2.1 recognize, extend, and create geometric, spatial, and numerical patterns;
2.2 solve problems in number theory, geometry, probability and statistics, and measurement and estimation using algebraic thinking;
2.3 communicate the meaning of variables in algebraic expressions and equations;
2.4 apply the concept of variable in simplifying algebraic expressions and solving equations;
2.5 interpret graphs that depict real-world phenomena;
2.6 model real-world phenomena using graphs.

Student Performance Indicators:

At Level 1, the student is able to

- extend geometric, spatial, and numeric patterns;
- generalize a variety of patterns with symbolic rules.

At Level 2, the student is able to

- represent situations and solve real-world problems using symbolic algebra;
- generate equivalent forms for simple algebraic expressions;
- solve one-step linear equations involving integers;
- evaluate a first-degree algebraic expression given values for up to two variables;
- apply given formulas to solve real-world problems;

At Level 3, the student is able to

- use a variety of representations to solve real-world problems (i.e., graphs, tables, equations);
- connect linear equations and the appropriate graphs of lines.
- explain the meaning of intercept, slope, and rate of change in real-world problems;

Sample Task: Use the Census Bureau website to find information regarding population patterns or economic data and display the results graphically.

Linkages: Write about patterns in nature or other areas of science, and find and apply formulas in business or science.

Content Standard 3.0: Geometry

Students will investigate, model, and apply geometric properties and relationships.

Learning Expectations:

The student will:

3.1 apply geometric properties, formulas, and relationships to solve real-world problems;
3.2 communicate position using spatial sense with two-dimensional coordinate systems;
3.3 demonstrate an understanding of the properties and construction of geometric figures, including angles, parallel lines, perpendicular lines, triangles, circles, and quadrilaterals;

Student Performance Indicators:

At Level 1, the student is able to

- use ordered pairs to describe given points in a coordinate system;
- apply the given formula to determine the area or perimeter of a rectangle;

At Level 2, the student is able to

- find the missing length of a side, given two similar triangles;
- classify a quadrilateral, given its properties, as a square, a rectangle, a rhombus, a parallelogram, and/or a trapezoid;
- calculate the area of a circle, a triangle, a parallelogram, a rhombus, or a trapezoid given the appropriate formula;

At Level 3, the student is able to

- apply the Pythagorean Theorem in problem solving;
- determine and justify the missing angle measures, given the measure of one angle, when two parallel lines are cut by a transversal.

Sample Task: Students use tessellating polygons to design a floor covering.

Linkages: Make connections of geometric concepts to careers such as design, art, and architecture.

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Content Standard 4.0 Measurement

Students will become familiar with the units and processes of measurement in order to use various tools, techniques, and formulas to determine and estimate measurements in problem solving.

Learning Expectations:

The student will:

4.1 apply appropriate techniques, tools, and formulas to determine measurements;
4.2 communicate the concepts and strategies being to estimate measurements;
4.3 apply measurement concepts and relationships in geometric problem-solving situations.

Student Performance Indicators:

At Level 1, the student is able to

- select units of appropriate size and type to measure angles, perimeter, area, surface area, and volume.

At Level 2, the student is able to

- use concepts of length, area, and volume to estimate and solve real- world problems (i. e., parallelograms, triangles, right rectangular prisms, circles, right cylinders);
- solve real-world problems involving rate/time/distance (i.e., d = rt);
- use estimation to make predictions and determine reasonableness of results.

At Level 3, the student is able to

- choose appropriate techniques and tools to measure quantities in order to meet specifications for precision and accuracy;
- estimate to find the area of irregular and complex shapes.

Sample Task: Students will measure and make a scale drawing for a room and determine the amount of carpet or tile needed and the amount of paint needed for the walls.

Linkages: Connect geometry and measurement concepts and relate measurement to construction, science, and other careers.

Content Standard 5.0 Data Analysis and Probability

Students will understand and apply basic statistical and probability concepts in order to organize and analyze data and to make predictions.

Learning Expectations:

The student will:

5.1 choose, construct, and analyze appropriate graphical representations for a data set including pie charts, histograms, stem and leaf plots, and scatterplots;
5.2 interpret a set of data using the appropriate measure of central tendency (mean, median, mode);
5.3 determine experimental and theoretical probabilities for simple experiments.

Student Performance Indicators:

At level 1, the student will be able to

- determine the mean, median, mode, and range for a given set of data;
- determine the probability of an event (i.e., rolling a die or spinning a spinner).

At Level 2, the student will be able to

- construct and interpret bar, circle, and line graphs of real-world data;
- determine the number of possible outcomes for simple experiments using lists, tree diagrams, or the multiplication counting principle;
- determine the median from a given stem-and-leaf plot.

At Level 3, the student will be able to

- make predictions based on data;
- develop meaning for lines of best fit.

Sample Task: Students will conduct and survey, graph the results, and use the results to make predictions.

Linkages: Connect analysis of data to examples from science, social studies, language arts, economics, politics, and other disciplines or careers. Use computer spreadsheets and graphing calculator lists to generate graphs of data sets.

Foundations II

Course Description:

Students will use problem situations, physical models, and appropriate technology to investigate concepts and topics that prepare them for higher level mathematics. Problem-solving situations will provide an environment that promotes communication and fosters connections within mathematics, to other disciplines, and to the real world. Students will use physical models to represent, explore, and develop abstract concepts. The use of appropriate technology will help students apply mathematics in an increasingly technological world.

Content Standard 1.0: Number and Operations

Students will recognize, represent, model, and apply real numbers and operations verbally, physically, symbolically, and graphically and will compute fluently and make reasonable estimates in problem solving.

Learning Expectations:

The student will:

1.1 demonstrate an understanding of the subsets, elements, properties, and operations of the real number system;
1.2 demonstrate an understanding of the relative size of rational and irrational numbers;
1.3 connect physical, graphical, verbal, and symbolic representations of real numbers;
1.4 informally describe and model the concept of inverse (e.g., opposites, reciprocals, and squares and square roots);
1.5 demonstrate an understanding of division involving zero;
1.6 describe, model, and apply inverse operations;
1.7 apply number theory concepts (e.g., primes, factors, divisibility and multiples) in mathematical problem situations;
1.8 connect physical, graphical, verbal, and symbolic representations of absolute value;
1.9 use real numbers to represent real-world applications (e.g., rate of change, probability, and proportionality);
1.10 select and apply an appropriate method (i.e., mental arithmetic, paper and pencil, or technology) for computing with real numbers, and evaluate the reasonableness of results;
1.11 communicate the concepts and strategies being used in estimation and computation;
1.12 perform operations on simple algebraic expressions, and informally justify the procedures chosen;
1.13 use estimation to make predictions and determine reasonableness of computational results;
1.14 use mathematical notations appropriately.

Performance Indicators State:

As documented through state assessment -

At Level 1, the student is able to

- choose the correct prime factorization of a two-digit composite whole number;
- compare a fraction to a decimal using less than, greater than, and equals symbols;
- multiply a fraction by a multiple of its denominator (denominator less than or equal to 25);
- apply order of operations to evaluate numerical expressions (whole numbers only; no exponents or grouping symbols.

At Level 2, the student is able to

- identify the opposite of any rational number;
- select the best estimate for the coordinate of a given point on a number line (rationals);
- choose an equivalent exponential form of a one-variable monomial given in factored form (only first-degree variables with positive integral coefficients);
- first-degree variables with positive integral coefficients);
- multiply an integer by a one-variable binomial;
- select a reasonable solution for a real-world division problem in which the remainder must be considered;
- apply order of operations to evaluate numerical expressions containing whole numbers, exponents, and no more than two sets of grouping symbols (no power larger than two);

at Level 3, the student is able to

- select ratios and proportions to represent real-world problems such as scale drawings and samplings (all ratios are positive integers to positive integers).

Performance Indicators Teacher:

As documented through teacher observation -

At Level 1, the student is able to

- classify numbers less than 100 as prime or composite;
- model rational numbers using manipulatives;
- classify a number as a whole number, an integer, a rational number, and/or a real number;
- select and apply the appropriate method for computing with real numbers (i.e., mental arithmetic, paper and pencil, or technology).

At Level 2, the student is able to

- compare a quotient when zero is the divisor to a quotient when zero is the dividend;
- relate sets of numbers using Venn diagrams;
- arrange a given set of rational numbers in ascending order;
- use concrete and pictorial representations to model the Distributive Property;
- connect a variety of real-world situations to integers;

At Level 3, the student is able to

- connect physical, graphical, verbal, and symbolic representations of absolute value;
- model inverse operations;
- explore various representations and equivalent forms of real numbers;
- justify, using models, operations on simple algebraic expressions (i.e., collecting like terms).

Sample Tasks: Search the newspapers for various uses of numbers and, in writing, justify the representations chosen (i.e., percent, fraction, and decimal). Investigate the applications of numbers and computations in the workplace.

Linkages: Mathematics - Estimation, Measurement, and Computation. Make connections to social studies through the study of latitude/longitude in mapping skills. Make connections to business/economics by tracking the stock market. Make connections to number representations and computations (e.g., interest) used in the workplace.

Standard 2.0: Algebra

Students will describe, extend, analyze, and create a wide variety of patterns and solve real-world problems using appropriate materials and representations.

Learning Expectations:

The student will:

2.1 recognize, extend, and create geometric, spatial, and numerical patterns;
2.2 analyze mathematical patterns related to algebra and geometry in real-world problem solving;
2.3 solve problems in number theory, geometry, probability and statistics, and measurement and estimation using algebraic thinking and symbolism (attention given to solving linear equations);
2.4 communicate the meaning of variables in algebraic expressions, equations, and inequalities;
2.5 interpret the results of algebraic procedures;
2.6 apply the concept of variable in simplifying algebraic expressions, solving equations, and solving inequalities;
2.7 interpret graphs that depict real-world phenomena;
2.8 model real-world phenomena using graphs.

Performance Indicators State:

As documented through state assessment -

At Level 1, the student is able to

- extend a pattern of geometric figures;
- extend a numerical pattern using only whole numbers.

At Level 2, the student is able to

- solve a one-step linear equation with a variable on only one side of the equation (integral coefficients and constants);
- solve a two-step linear equation with a variable on only one side of the equation (integral coefficients
- and constants);
- translate a one-variable verbal expression into an algebraic expression (no more than two operations);
- evaluate a first-degree algebraic expression given the values for the variables (up to three variables);
- select the appropriate linear graph that models a real-world situation.

At Level 3, the student is able to

- select the number line graph that models a given one-step linear inequality (variables may not have negative coefficients);
- simplify a first-degree algebraic expression by combining like terms (integral coefficients and constants).

Performance Indicators Teacher:

As documented through teacher observation -

At Level 1, the student is able to

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- complete a function table given the function rule.

At Level 2, the student is able to

- construct linear and non-linear graphs that model given real-world situations;
- solve a two-step linear equation using models and justify each step.

At Level 3, the student is able to

- use the Distributive Property to solve a one-variable linear equation (variable on both sides of the equation);
- solve and graph linear inequalities with integral coefficients.

Sample Tasks: Use the internet or daily newspaper to find information regarding the stock results of one to five companies. Graphically demonstrate the histories of the companies for five to thirty days. Using the information found, predict the status of the company five to thirty days from now. Develop a range of the companies' stock values for the last five to thirty days, using inequalities. Write a scenario explaining why a company's stock changed drastically.

Linkages: Statistics and Probability. Trends in finance and in the business world. Science - numerical patterns in human anatomy (golden ratio)

Standard 3.0: Geometry

Students will investigate, model, and apply geometric properties and relationships.

Learning Expectations:

The student will:

3.1 analyze relationships among corresponding parts of similar or congruent geometric figures;
3.2 apply geometric properties, formulas, and relationships to solve real-world problems;
3.3 use inductive reasoning to make conjectures;
3.4 communicate position using spatial sense with two-dimensional coordinate system;
3.5 demonstrate an understanding of transformations of geometric figures;
3.6 apply the Pythagorean Theorem in problem solving;
3.7 name, analyze, and describe the properties of various polygons.

Performance Indicators State:

As documented through state assessment -

At Level 1, the student is able to

- determine the perimeter of any geometric figure.

At Level 2, the student is able to

- identify the coordinates for a given point;
- find the missing length of a side given two similar triangles.

At Level 3, the student is able to

- use the Pythagorean Theorem to determine the length of a missing side of a right triangle (no radicals).

Performance Indicators Teacher:

As documented through teacher observation -

At Level 1, the student is able to

- identify corresponding parts of congruent triangles;
- classify a quadrilateral, given its properties, as a square, a rectangle, a rhombus, a parallelogram, and/or a trapezoid.

At Level 2, the student is able to

- model a variety of triangles (i.e., acute, obtuse, scalene, isosceles, and equilateral);
- use the Triangle Inequality to determine if given lengths could form a triangle;
- calculate the area of a circle, a triangle, a parallelogram, a rhombus, or a trapezoid given the appropriate formula.

At Level 3, the student is able to

- classify a variety of polygons then justify the classification;
- determine and justify the missing angle measures, given the measure of one angle, when two parallel lines are cut by a transversal.

Sample Tasks: Students choose tessellating polygon shapes to design a quilt pattern. Students use the properties of similar triangles to investigate the height of trees, flagpoles, and other structures.

Linkages: Construction, Design, Architecture, and Art

Standard 4.0: Measurement

Students will become familiar with the units and processes of measurement in order to use various tools, techniques, and formulas to determine and estimate measurements in problem solving.

Learning Expectations:

The student will:

4.1 communicate the concepts and strategies used to measure and to estimate measurements;
4.2 use concepts of length and area, including surface area and volume, to estimate and solve real-world problems (e.g., parallelograms, triangles, right rectangular prisms, circles, right cylinders, spheres, and pyramids);
4.3 apply measurement concepts and relationships in algebraic and geometric problem-solving situations;
4.4 choose appropriate techniques and tools to measure quantities in order to meet specifications for precision and accuracy;
4.5 demonstrate an understanding of rates and other derived and indirect measurements (e.g., velocity, miles per hr, rpm, and cost per unit).

Performance Indicators State:

As documented through state assessment –

At Level 1, the student is able to

- apply the given formula to determine the area of a rectangular figure with rational dimensions.

At Level 2, the student is able to

- calculate the cost per unit to determine the best buy (no more than three samples).

At Level 3, the student is able to

- choose the correct area representation of the product of an integer and a one-variable first-degree binomial.

Performance Indicators Teacher:

As documented through teacher observation -

At Level 1, the student is able to

- measure a line segment using appropriate metric or customary units;
- choose an appropriate unit of measure in a real-world situation (e.g., length of a car, length of a pencil, or weight of a person).

At Level 2, the student is able to

- apply the concept of rate to determine ones such as mph, cost per unit, rpm.

At Level 3, the student is able to

- model and compare the area and dimensions of similar polygons (i.e., triangles, squares, and rectangles).

Sample Tasks: Provide students a scale drawing of a room and have them calculate the amount and cost of various floor-covering. Give students a blow pop and have them measure the circumference. Have students rotate the blow pop in their mouths for two minutes. Measure the circumference again. Repeat this process five times. Plot time versus circumference and explain the change.

Linkages: Mathematics – Geometry and Science. Connect to careers such as auto mechanic, construction, travel, etc.

Standard 5.0: Data Analysis and Probability

Students will interpret a given set of data, including analyzing the use, misuse, and abuse of data; choose, construct, and analyze appropriate graphical representations for a data set; use technology in data collection and analysis; and apply theoretical and experimental probability to analyze the likelihood of an event.

Learning Expectations:

The student will:

5.1 interpret a set of data using the appropriate measure of central tendency (mean, median, mode) and the appropriate measure of dispersion (e.g., quartiles, range);
5.2 choose, construct, and analyze appropriate graphical representations for a data set including pie charts, histograms, stem-and-leaf plots, scatterplots, and box plots;
5.3 apply appropriate technology in data collection and analysis;
5.4 apply theoretical and experimental probability to analyze the likelihood of an event;
5.5 use simulations to estimate probability;
5.6 analyze the validity of statistical conclusions and the use, misuse, and abuse of data;
5.7 apply counting principles of permutations and combinations using appropriate technology.

Performance Indicators State:

As documented through state assessment -

At Level 1, the student is able to

- determine the mean of a given set of data (no more than five two-digit numbers);
- determine the number of possible outcomes for a simple experiment using a list, tree diagram, or the multiplication counting principle;
- determine the probability of a single event (i.e. rolling a die and using a spinner).

At Level 2, the student is able to

- interpret bar graphs representing real-world data;
- interpret circle graphs (pie charts) representing real-world data;
- determine the median from a given stem-and-leaf plot.

At Level 3, the student is able to

- determine the median of a given set of real-world data (even number of data);
- select the measure of central tendency that best describes the given real-world situation.

Performance Indicators Teacher:

As documented through teacher observation -

At Level 1, the student is able to

- construct a bar graph using real-world data;
- construct a circle graph using real-world data;
- construct a line graph using real-world data.

At Level 2, the student is able to

- construct a stem-and-leaf plot using real-world data.

At Level 3, the student is able to

- use simulations to estimate probability.

Sample Tasks: Survey 100 people on political candidates, favorite shows, etc. Create a frequency table and display the data. Randomly sample the school population and survey those students to predict the opinions of other students on certain topics.

Linkages: Technology - Students create spreadsheets and graphs on the computer and then decide which type of graph best displays their data. Social Studies - Census Bureau, Neilson Ratings, National Polls.

Technical Mathematics (Mathematics for Technology I)

Course Description:

Technical Mathematics uses problem situations, physical models, and appropriate technology to extend mathematical thinking and engage student reasoning. Problem solving situations will provide all students an environment that promotes communication and fosters connections within mathematics, to other disciplines, and to the technological workplace. Students will use physical models in a laboratory setting to represent, explore, and develop abstract concepts. The use of appropriate technology will help students apply mathematics in an increasingly technological world. The course includes: problem solving, reasoning, connections, communication, and representation and the relationship of these mathematical processes to applications of the content in the workplace.

Content Standard 1.0: Number and Operations

Students will recognize, represent, model, and apply real numbers and operations verbally, physically, symbolically, and graphically and will compute fluently and make reasonable estimates in problem solving related to the workplace.

Learning Expectations:

The student will:

1.1 demonstrate an understanding of the subsets, elements, properties, and operations of the real number system;
1.2 demonstrate an understanding of the relative size of rational and irrational numbers;
1.3 connect physical, graphical, verbal, and symbolic representations of real numbers;
1.4 informally describe and model the concept of inverse (e.g., opposites, reciprocals, and squares and square roots);
1.5 demonstrate an understanding of division involving zero;
1.6 apply number theory concepts (e.g., primes, factors, divisibility and multiples) in mathematical problem situations;
1.7 connect physical, graphical, verbal, and symbolic representations of absolute value;
1.8 use real numbers to represent real-world applications (e.g., rate of change, probability, and proportionality);
1.9 select and apply an appropriate method (i.e., mental arithmetic, paper and pencil, or technology) for computing with real numbers, and evaluate the reasonableness of results;
1.10 perform operations on simple algebraic expressions, and informally justify the procedures chosen;
1.11 use estimation to make predictions and determine reasonableness of computational results;
1.12 use mathematical notations appropriately.

Student Performance Indicators:

At Level 1, the student is able to

- choose the correct prime factorization of a two-digit composite whole number;
- compare a fraction to a decimal using less than, greater than, and equals symbols;
- multiply a fraction by a multiple of its denominator (denominator less than or equal to 25);
- apply order of operations to evaluate numerical expressions (whole numbers only; no exponents or grouping symbols.

At Level 2, the student is able to

- identify the opposite of any rational number;
- select the best estimate for the coordinate of a given point on a number line (rationals);
- choose an equivalent exponential form of a one-variable monomial given in factored form (only first-degree variables with positive integral coefficients);
- multiply an integer by a one-variable binomial;
- select a reasonable solution for a real-world division problem in which the remainder must be considered;
- apply order of operations to evaluate numerical expressions containing whole numbers, exponents, and no more than two sets of grouping symbols (no power larger than two);

at Level 3, the student is able to

- select ratios and proportions to represent real-world problems such as scale drawings and samplings (all ratios are positive integers to positive integers).

Content Standard 2.0: Algebra

Students will describe, extend, analyze, and create a wide variety of patterns and functions using appropriate materials and representations in real world problem solving.

Learning Expectations:

The student will:

2.1 analyze, extend, and create mathematical patterns related to algebra in real-world problem solving;
2.2 communicate the meaning of variables in algebraic expressions and equation;
2.3 apply and interpret rates of change from numerical data;
2.4 apply the concept of variable to simplify algebraic expressions and solve equations;
2.5 model real-world phenomenon using graphs.

Student Performance Indicators:

At Level 1, the student is able to

- describe and extend geometric and numerical patterns in the workplace.

At Level2, the student is able to

- translate a verbal expression into an algebraic expression in real-world problems;
- evaluate real-world formulas and algebraic expressions given values for one or more variables and grouping symbols;
- solve one- and two-step linear equations;
- justify correct results of algebraic procedures.

At Level 3, the student is able to

- apply the concept of rate of change to solve real-world problems;
- select the linear and non-linear graphs that model given real-world situations described in data sets and narratives.

Standard 3.0: Geometry

The student will investigate, model, and apply geometric properties and relationship in work related situations.

Learning Expectations:

The student will:

3.1 analyze and apply concepts and properties in the construction of lines, angles, and vertices when solving work-related problems;
3.2 synthesize and apply geometrical concepts, properties, and formulas of two-dimensional shapes when solving work-related problems;
3.3 synthesize and apply geometrical concepts, properties, and formulas of three-dimensional shapes when solving work-related problems.

Student Performance Indicators:

At Level 1, the student will be able to

- construct angles and vertices to solve work-related problems.

At Level 2, the student will be able to

- use geometric formulas to solve real-world problems (e.g. area, perimeter, surface area, volume and circumference);
- use scale and proportion in real-world situations (e.g. read a road map, scale drawings, and read blueprints);
- apply the Pythagorean Theorem in real-world problems;
- determine the height of an object that is difficult to measure by using properties of similar triangles.

At Level 3, the student will be able to

- use parallel and perpendicular lines to solve work-related problems.

Standard 4.0: Measurement

Students will become familiar with the units and processes of measurement in order to use various tools, techniques, and formulas to determine and estimate measurements in problem solving.

Learning Expectations:

The student will:

4.1 select and use appropriate tools of measurement to determine length, area, angular measurement and volume with in given tolerances (i.e. vernier caliper, micrometer, machinist rule, graduated cylinders, protractors as well as rulers);
4.2 use measurements of length, area, angular measurement and volume to estimate and solve real-world problems;
4.3 apply measurement concepts and relationships in algebraic and geometric problem-solving situations;
4.4 demonstrate an understanding of rates and other derived and indirect measurements.

Student Performance Indicators

At Level 1, the student is able to

- select and apply appropriate tools and units to measure in real-world situations (e.g., manufacturing, construction, art);
- justify the selection of a unit of measure in specific situations (e.g., manufacturing);
- discover and explain formulas used to compute circumference, perimeter, area and volume (e.g., pool construction);
- apply the given formula to determine the area, perimeter or volume of two dimensional objects;
- defend estimates of the perimeter and/or area of rectangles, triangles, trapezoids, and parallelograms. (e.g., flooring);
- estimate the area and volume of irregular geometric figures in work-related problems.

At Level 2, the student is able to

- apply the given formula to find the area of a circle, the circumference of a circle, or the volume and surface area of a rectangular solid, cylinder, and sphere;
- apply the concept of rate to determine ones such as miles per hour, cost per unit, and revolutions per minute;
- describe the procedure for determining the area of a composite shape in a real-world situation (e.g., surveying);
- defend an estimate for the volume of a container (e.g. bottling companies);
- compare various methods of measurement to estimated values (e.g., shadow of object vs. height of object);
- calculate a dimension of a geometric figure given the volume and other pertinent information (e.g. housing);
- determine if measurements are within given tolerance intervals;
- construct scale drawings to solve work related problems.

at Level 3, the student is able to

- discover the dimensions of a rectangle when given its area and the relationship between the length and width of the sides (e.g., art);
- explore the golden rectangle as it relates to measurement and proportions
- describe how changes in the dimensions of similar figures affect perimeter, area, and volume (e.g., construction).

Sample Task: Students will measure and make a scale drawing for a room and determine the amount of carpet or tile needed and the amount of paint needed for the walls.

Linkages: Mathematics – Geometry. Discuss connections to drafting and carpentry, agribusiness, marketing, consumer science, and industrial technology.

Standard 5.0: Data Analysis and Probability

The student will collect, organize, represent, and interpret data; make and evaluate inferences and predictions; present and evaluate arguments based on data analysis; and model situations to determine theoretical and experimental probabilities.

Learning Expectations:

The student will:

5.1 read graphs, charts, and tables;
5.2 recognize if a problem needs more data and if so find a source for the data;
5.3 collect, organize and interpret data;
5.4 choose, construct, and analyze appropriate graphical representations for a data set;
5.5 interpret a set of data using the appropriate measure of central tendency;
5.6 interpolate readings on a graph as well as extrapolate to estimate values;
5.7 apply appropriate technology in data collection and analysis;
5.8 analyze the validity of statistical conclusions and the use, misuse, and abuse of data.

Student Performance Indicators:

At Level 1, the student is able to

- interpret bar graphs representing real-world data;
- interpret circle graphs representing real-world data;
- determine the measures of central tendency for a given set of data;
- determine the probability of a single event (i.e., spinning a spinner, rolling a die);
- collect data from a real-world situation and construct a graph (bar, circle, line) both by hand and using appropriate technology.

At Level 2, the student is able to

- choose the matching linear graph given a set of ordered pairs that represent real-world data;
- analyze student-collected data from a real-world situation to make predictions using appropriate technology;
- apply the appropriate measure of central tendency (i.e., mean, median, and mode) to a real-world problem.

At Level 3, the student is able to

- select the measure of central tendency that best describes the given real-world situation;
- choose the matching scatter plot, bar graph, or histogram given a set of real-world data in table or chart form;
- choose the correlation of a scatter plot using real-world data; analyze the validity of statistical conclusions and the use, misuse, and abuse of data;
- use simulations to determine probabilities.

Sample Task: Students conduct a consumer preference survey and construct a graph to show the results and make any appropriate predictions.

Linkages: Students create spreadsheets and graphs on the computer to decide what type of graph best displays their data. Economics – Census Bureau, Neilson Ratings. Sports – game statistics.

Algebra I

Course Description:

Algebra I uses problem situations, physical models, and appropriate technology to extend algebraic thinking and engage student reasoning. Problem solving situations will provide all students an environment that promotes communication and fosters connections within mathematics, to other disciplines and to the real world. Students will use physical models to represent, explore, and develop abstract concepts. The use of appropriate technology will help students apply mathematics in an increasingly technological world. The concepts emphasized in the course include functions, solving equations, slope as rates of change, and proportionality.

Standard 1.0: Number and Operations

Students will recognize, represent, model, and apply real numbers and operations verbally, physically, symbolically, and graphically.

Learning Expectations:

The student will:

1.1 demonstrate an understanding of the subsets, properties, and operations of the real number system;
1.2 demonstrate an understanding of the relative size of rational and irrational numbers;
1.3 articulate, model, and apply the concept of inverse (e.g., opposites, reciprocals, and powers and roots);
1.4 describe, model, and apply inverse operations;
1.5 apply number theory concepts (e.g., primes, factors, divisibility and multiples) in mathematical problem solving;
1.6 connect graphical and symbolic representations of absolute value;
1.7 use real numbers to represent real-world applications (e.g., slope, rate of change, probability, and proportionality);
1.8 use a variety of notations appropriately (e.g. exponential, functional, square root);
1.9 select and apply an appropriate method (i.e., mental mathematics, paper and pencil, or technology) for computing with real numbers, and evaluate the reasonableness of results;
1.10 perform operations on algebraic expressions and informally justify the procedures chosen;
1.11 perform operations on matrices in real-world problem solving (i.e., addition, subtraction, and scalar multiplication).

Performance Indicators State:

As documented through state assessment –

At Level 1, the student is able to

- select the best estimate for the coordinate of a given point on a number line (only rational);
- identify the opposite of a rational number;
- determine the square root of a perfect square less than 169;
- use exponents to simplify a monomial written in expanded form;
- apply order of operations when computing with integers using no more than two sets of grouping symbols and exponents 1 and 2;
- select a reasonable solution for a real-world division problem in which the remainder must be considered.

At Level 2, the student is able to

- order a given set of rational numbers (both fraction and decimal notations);
- identify the reciprocal of a rational number;
- add and subtract algebraic expressions;
- multiply two polynomials with each factor having no more than two terms;
- use estimation to determine a reasonable solution for a tedious arithmetic computation;
- select ratios and proportions to represent real-world problems (e.g. scale drawings, sampling, etc.).

At Level 3, the student is able to

- apply the concept of slope to represent rate of change in a real-world situation.

Performance Indicators Teacher:

As documented through teacher observation –

At Level 1, the student is able to

- connect a variety of real-world situations to integers;
- use manipulatives to represent commutative and associative properties of addition and multiplication;
- investigate alternate algorithms that show the relationship of division to subtraction and multiplication to addition;
- analyze prime and composite numbers;
- compare and contrast the GCF and LCM of a set of numbers;
- refine strategies for estimating whole numbers, fractions, and percentages.

At Level 2, the student is able to

- probe the relationships among various subsets of the real number system;
- compare and contrast the GCF and LCM of a set of algebraic expressions;
- construct a number line to describe the absolute value of a number as distance from zero;
- model operations using real-world situations and physical representations;
- perform operations on matrices using appropriate technology (addition, subtraction, and scalar multiplication);
- explore various representations of absolute value.

At Level 3, the student is able to

- research the history of prime numbers and their uses;
- scrutinize approximate values of real numbers such as pi and the square root of two.

Sample Tasks: Students design a concept map that illustrates the relationship among decimals, fractions, and percents. Students summarize in writing their concept maps and discuss how equivalent fractions, decimals, and percents can be flexibly interchanged. Students justify the selection of fraction, decimal, or percent notation in specific situations.

Linkages: Mathematics - Estimation, Measurement, and Computation. Make connections to scientific notation used in science, social studies, and finance.

Standard 2.0: Algebra

Students will describe, extend, analyze, and create a wide variety of patterns and functions using appropriate materials and representations in real world problem solving.

Learning Expectations:

The student will:

2.1 recognize, analyze, extend, and create a variety of patterns;
2.2 use algebraic thinking to generalize a pattern by expressing the pattern in functional notation;
2.3 solve linear systems using a variety of techniques;
2.4 communicate the meaning of variables in algebraic expressions, equations, and inequalities;
2.5 identify and represent a variety of functions;
2.6 apply and interpret rates of change from graphical and numerical data;
2.7 analyze graphs to describe the behavior of functions;
2.8 interpret results of algebraic procedures;
2.9 apply the concept of variable in simplifying algebraic expressions, solving equations, and solving inequalities;
2.10 interpret graphs that depict real-world phenomena;
2.11 model real-world phenomena using functions and graphs;
2.12 articulate and apply algebraic properties in symbolic manipulation;
2.13 analyze relationships which can and which cannot be represented by a function;
2.14 graph inequalities and interpret graphs of inequalities;
2.15 describe the domain and range of functions and articulate restrictions imposed either by the operations or by the real-life situations which the functions represent;
2.16 describe the transformation of the graph that occurs when coefficients and/or constants of the corresponding linear equations are changed.

Performance Indicators State:

As documented through state assessment –

At Level 1, the student is able to

- extend a geometric pattern;
- extend a numerical pattern;
- translate a verbal expression into an algebraic expression;
- evaluate a first degree algebraic expression given values for one or more variables;
- solve one- and two-step linear equations using integers (with integral coefficients and constants).

At Level 2, the student is able to

- select the algebraic notation which generalizes the pattern represented by data in a given table;
- translate a verbal sentence into an algebraic equation;
- select the graph that represents a given linear function expressed in slope-intercept form;
- solve multi-step linear equations (more than two steps, variables on only one side of the equation);
- solve multi-step linear equations (more than two steps, with variables on both sides of the equation);

- solve multi-step linear equations (more than two steps, with one set of parentheses on each side of the equation);
- select the linear graphs that models the given real-world situation described in a narrative (no data set given);
- select the linear graph that models the given real-world situation described in a tabular set of data;
- evaluate an algebraic expression given values for one or more variables using grouping symbols and/or exponents less than four;
- determine the slope from the graph of a linear equation (no labeled points);
- apply the concept of rate of change to solve real-world problems;
- select the appropriate graphical representation of a given linear inequality;
- select the non-linear graph that models the given real-world situation or vice versa;
- identify the graphical representation of the solution to a one variable inequality on a number line.

At Level 3, the student is able to

- solve multi-step linear inequalities in real-world situations;
- recognize the graphical transformation that occurs when coefficients and/or constants of the corresponding linear equations are changed;
- determine the domain and/or range of a function represented by the graph of real-world situations;
- select the system of equations that could be used to solve a given real-world problem; *
- find the solution to a quadratic equation given in standard form (integral solutions and a leading coefficient of one); *
- select the solution to a quadratic equation given solutions represented in graphical form (integral solutions and a leading coefficient of one); *
- select one of the factors (x + 3) of a quadratic equation (integral solutions and a leading coefficient of one); *
- select the discriminant of a quadratic equation (integral solutions and a leading coefficient of one). *

* Recommended by the 2003 committee as additional state performance indicators. Additional state performance indicators will begin to be assessed during 2005-2006.

Performance Indicators Teacher:

As documented through teacher observation –

At Level 1, the student is able to

- analyze rational number patterns;
- describe in writing the pattern for real-world data listed in a function table.

At Level 2, the student is able to

- produce an equation to describe the relationship between data sets;
- explore patterns including Pascal's Triangle and a Fibonacci sequence;
- solve a system of two linear equations using the graphing, elimination, and substitution methods;
- defend the selection of a method for solving a system of equations;
- represent algebraic expressions and operations using manipulatives;
- model the steps for solving simple linear equations using manipulatives;
- write an equation that symbolically expresses a problem solving situation;

- justify correct results of algebraic procedures;
- distinguish between a function and other relationships.

At Level 3, the student is able to

- analyze "families of functions" using technology;

Sample Tasks: Use an almanac or the internet to find the area and the average depth of the world's ten largest bodies of salt water. Draw a scatterplot showing the relationship between these two sets of data. Describe the relationship and determine if it is a functional relationship.

Links: Mathematics - Statistics and Probability. Have students recognize the use of patterns in other disciplines and in a variety of cultures

Standard 3.0: Geometry

The student will investigate, model, and apply geometric properties and relationships.

Learning Expectations:

The student will:

3.1 apply geometric properties, formulas, and relationships to solve real-world problems;
3.2 solve problems using the midpoint formula;
3.3 apply right triangle relationships including the Pythagorean Theorem and the distance formula;
3.4 find and represent solutions of quadratic equations geometrically.

Performance Indicators State:

As documented through state assessment –

At Level 1, the student is able to

- identify ordered pairs in the coordinate plane.

At Level 2, the student is able to

- apply the given Pythagorean Theorem to a real life problem illustrated by a diagram (no radicals in answer);
- apply proportion and the concepts of similar triangles to find the length of a missing side of a triangle.

At Level 3, the student is able to

- calculate the distance between two points given the Pythagorean Theorem and the distance formula.

Performance Indicators Teacher:

As documented through teacher observation –

At Level 1, the student is able to

- describe real-world uses of geometric formulas and relationships
- discuss issues related to estimating areas of irregular-shaped figures for real-world uses (i.e. fencing, painting, laying carpet, purchasing wallpaper or border).

At Level 2, the student is able to

- explain how to determine if a triangle is a right triangle given the measurements of all three sides;
- illustrate the Pythagorean Theorem by measuring the length, width, and diagonals of rectangular objects; design area models to illustrate the Pythagorean Theorem.

At Level 3, the student is able to

- determine the height of an object that is difficult to measure by using the properties of similar triangles.

Sample Task: Approximate the value of pi (π) by looking at the relationship between the diameter and circumference of various circular objects after measuring using a string or a tape measure. Students research and write about how various geometric properties are used in careers such as construction, drafting, and surveying.

Linkages: Mathematics - Estimation, Measurement, and Computation, Research, and the geometric applications in art.

Standard 4.0: Measurement

The student will apply appropriate tools and units of measurement to produce reasonable results.

Learning Expectations:

The student will:

4.1 use concepts of length, area, and volume to estimate and solve real-world problems;
4.2 apply and communicate measurement concepts and relationships in algebraic and geometric problem-solving situations;
4.3 demonstrate an understanding of rates and other derived and indirect measurements (e.g., velocity, miles per hour, revolutions per minute, cost per unit);
4.4 make decisions about units, scales, and measurement tools that are appropriate for problem situations involving measurement;
4.5 analyze precision, accuracy, tolerance, and approximate error in measurement situations.

Performance Indicators State:

As documented through state assessment –

At Level 1, the student is able to

- estimate the area of irregular geometric figures on a grid;
- calculate rates involving cost per unit to determine the best buy (no more than three samples)'
- apply the given formula to determine the area or perimeter of a rectangle.

At Level 2, the student is able to

- apply the given formula to find the area of a circle, the circumference of a circle, or the volume of a rectangular solid.

At Level 3, the student is able to

- select the area representation for a given product of two one-variable binomials with positive constants and coefficients.

Performance Indicators Teacher:

As documented through teacher observation –

At Level 1, the student is able to

- justify the selection of a unit of measure in specific situations;
- defend estimates of the perimeter and/or area of rectangles and triangles;
- discover and explain formulas used to compute area and volume.

At Level 2, the student is able to

- describe the procedure for determining the area of a composite shape in a real-world situation;
- generalize area formulas using manipulatives for a parallelogram, a triangle, and a trapezoid;
- defend an estimate for the volume of a container;
- relate the volume of a container to its shape;
- analyze precision, accuracy, tolerance, and approximate error in measurement situations.

At Level 3, the student is able to

- discover the dimensions of a rectangle when given its area and the relationship between two adjacent sides;
- describe how changes in the dimensions of figures affect perimeter, area, and volume.

Sample Task: Place students in small groups giving each group a different length of string. Have each group form a rectangle with the string. Ask each group to measure the sides of their rectangle and find its area. Using the string, direct each group to construct the rectangle with the greatest possible area. Give each group the opportunity to justify their solution.

Linkages: Mathematics – Geometry. Use formulas in Science. Discuss connections to drafting and carpentry. Connect estimation and computation strategies to business and finance.

Standard 5.0: Data Analysis and Probability

The student will collect, organize, represent, and interpret data and model situations to determine theoretical and experimental probabilities.

Learning Expectations:

The student will:

5.1 collect, represent, and describe linear and nonlinear data sets developed from the real world;
5.2 make predictions from a linear data set using a line of best fit;

5.3 interpret a set of data using the appropriate measure of central tendency;
5.4 choose, construct, and analyze appropriate graphical representations for a data set;
5.5 understand the concept of random sampling;
5.6 apply counting principles of permutations and combinations using appropriate technology;
5.7 model situations to determine theoretical and experimental probabilities.

Performance Indicators State:

As documented through state assessment –

At Level 1, the student is able to

- determine the mean (average) of a given set of real-world data (no more than five two-digit numbers);
- interpret bar graphs representing real-world data;
- interpret circle graphs (pie charts) representing real-world data.

At Level 2, the student is able to

- choose the matching linear graph given a set of ordered pairs;
- make a prediction from the graph of a real-world linear data set;
- determine the median for a given set of real-world data (even number of data).

At Level 3, the student is able to

- apply counting principles of permutations or combinations in real-world situations.

Performance Indicators Teacher:

As documented through teacher observation –

At Level 1, the student is able to

- design a strategy for collecting real-world data for a scientific investigation;
- collect and organize real-world data.

At Level 2, the student is able to

- graph real-world data using a variety of representations;
  debate the selection of a graphical representation which best describes specific data;
- model situations to determine theoretical and experimental probabilities;
- judge the validity of claims made in probabilistic situations;
- defend the sampling method chosen to conduct a survey.

At Level 3, the student is able to

- debate possible conclusions that can be supported by the data;
- make predictions from real-world data using a line of best fit.

Sample Task: Students research the age of each Tennessee governor at the time of his/her inauguration. The students organize their information and will determine which measure of central tendency is the best description of the data. Students explain their decision.

Linkages: Mathematics - Patterns, Functions, and Algebraic Thinking. Analyze census data. Research and discuss the careers that require the use of statistics such as statistician, actuaries, and scientists.

Technical Algebra (Mathematics for Technology II)

Course Description:

Technical Algebra uses problem situations, physical models, and appropriate technology to extend algebraic thinking and engage student reasoning. Problem solving situations, including those related to a variety of careers and technical fields, will provide all students an environment which promotes communication and fosters connections within mathematics, to other disciplines and to the technological workplace. Students will use physical models in a laboratory setting to represent, explore, and develop abstract concepts. The use of appropriate technology will help students apply algebra in an increasingly technological world. The concepts emphasized in the course include: measurement, statistical data analysis, functions, solving equations, and slope as rates of change, and proportionality.

Standard 1.0: Number and Operations

Students recognize, represent, model, and apply real numbers and operations verbally, physically, symbolically, and graphically.

Learning Expectations:

The student will:

1.1 demonstrate an understanding of the subsets, properties, and operations of the real number system;
1.2 demonstrate an understanding of the relative size of rational and irrational numbers;
1.3 articulate, model, and apply the concept of inverse (opposites and reciprocals, and powers and roots);
1.4 describe, model, and apply inverse operations;
1.5 apply number theory concepts (, primes, factors, divisibility and multiples) in mathematical problem solving;
1.6 connect graphical and symbolic representations of absolute value;
1.7 use real numbers to represent real-world applications (, slope, rate of change, probability, and proportionality);
1.8 use a variety of notations appropriately (exponential, functional, square root);
1.9 select and apply an appropriate method (e.g., mental mathematics, paper and pencil, or technology) for computing with real numbers, and evaluate the reasonableness of results;
1.10 perform operations on algebraic expressions and informally justify the procedures chosen;
1.11 perform operations on matrices in real-world problem solving situations, (i.e. addition, subtraction and scalar multiplication).

Performance Indicators State:

As documented through state assessment –

At Level 1, the student is able to

- select the best estimate for the coordinate of a given point on a number line (only rational);
- identify the opposite of a rational number;
- determine the square root of a perfect square less than 169;
- use exponents to simplify a monomial written in expanded form;
- apply order of operations when computing with integers using no more than two sets of grouping symbols and exponents 1 and 2;
- select a reasonable solution for a real-world division problem in which the remainder must be considered.

At Level 2, the student is able to

- order a given set of rational numbers (both fraction and decimal notations);
- identify the reciprocal of a rational number;
  add and subtract algebraic expressions;
- multiply two polynomials with each factor having no more than two terms;
- use estimation to determine a reasonable solution for a tedious arithmetic computation;
- select ratios and proportions to represent real-world problems (i.e., scale drawings, sampling).

At Level 3, the student is able to

- apply the concept of slope to represent rate of change in a real-world situation.

Performance Indicators Teacher:

As documented through teacher observation –

At Level 1, the student is able to

- connect a variety of real-world situations to integers (e.g., sports);
- use manipulatives to represent commutative and associative properties of addition and multiplication (e.g., lumber industry, board feet);
- investigate alternate algorithms that show the relationship of division to subtraction and multiplication to addition (e.g., accounting);
- analyze prime and composite numbers (e.g., masonry, tessellations);
- compare and contrast the GCF and LCM of a set of numbers (e.g., pattern layouts, manufacturing).

At Level 2, the student is able to

- probe the relationships among various subsets of the real number system (e.g., wildlife management, which set of animals are harvested or categorized);
- compare and contrast the GCF and LCM of a set of algebraic expressions (e.g., construction, by changing the width of patio blocks "w" how do you get the blocks to same dimensions as an existing patio who's width is 2.5 "w"?);
- construct a number line to describe the absolute value of a number as distance from zero (e.g., search and rescue team, how far east and west could a lost student be in "x" numbers of minutes);
- model operations using real-world situations and physical representations (e.g., medical field, establishing correct dosages from a formula);
- perform operations on matrices in real-world problem solving situations using technology (i.e. addition, subtraction and scalar multiplication; e.g., manufacturing);
- explain the importance of the value of the determinant of a matrix (e.g., systems problems in packaging);
- explore various representations of absolute value (e.g., auto body, restoring alignment of the frame after an accident).

At Level 3, the student is able to

- research the history of pi and its usages in the real world (e.g. effect of tire size on an odometer);
- use technology to solve systems of equations using matrices (e.g., manufacturing);
- scrutinize approximate values of real numbers such as pi and other irrational numbers (e.g., landscaping amount of edging needed for a circular flower garden).

Sample Tasks: Students use the exponential growth and decay models to explore the effects and decimal values used in the formula. They compare and contrast various rates of increase and decrease and discover the effect of the changes in the model of the graph of the function and its table of values. Sudents will graph the tolerances of work-related problems generated from absolute value functions. The students should be able to identify upper and lower levels of tolerances.

Linkages: Mathematics - Estimation, Measurement, and Computation. Make connections to scientific notation used in science, social studies, and finance, agribusiness, marketing, consumer science and industrial technology. Connect estimation and computation strategies to business and finance, construction.

Standard 2.0: Algebra

Students describe, extend, analyze, and create a wide variety of patterns and functions using appropriate materials and representations in real world problem solving.

Learning Expectations:

The student will:

2.1 recognize, analyze, extend, and create a variety of patterns;
2.2 use algebraic thinking to generalize a pattern by expressing the pattern in functional notation;
2.3 communicate the meaning of variables in algebraic expressions, equations, and inequalities;
2.4 identify and represent a variety of functions using technology;
2.5 apply and interpret rates of change from graphical and numerical data
2.6 analyze graphs to describe the behavior of functions;
2.7 interpret results of algebraic procedures;
2.8 apply the concept of variable in simplifying algebraic expressions, solving equations, and solving inequalities;
2.9 interpret graphs that depict real-world phenomena;
2.10 model real-world phenomena using functions and graphs;
2.11 articulate and apply algebraic properties in symbolic manipulation;
2.12 analyze relationships which can and which cannot be represented by a function;
2.13 graph inequalities and interpret graphs of inequalities;
2.14 describe the domain and range of functions and articulate restrictions imposed either by the operations or by the real-life situations which the functions represent;
2.15 describe the transformation of the graph that occurs when coefficients and/or constants of the corresponding linear equations are changed.

Performance Indicators State:

As documented through state assessment –

At Level 1, the student is able to

- extend a geometric pattern;
- extend a numerical pattern;
- translate a verbal expression into an algebraic expression;
- evaluate a first degree algebraic expression given values for one or more variables;

- solve one- and two-step linear equations using integers (with integral coefficients and constants).

At Level 2, the student is able to

- select the algebraic notation which generalizes the pattern represented by data in a given table;
  translate a verbal sentence into an algebraic equation;
- select the graph that represents a given linear function expressed in slope-intercept form;
- solve multi-step linear equations (more than two steps, variables on only one side of the equation);
- solve multi-step linear equations (more than two steps, with variables on both sides of the equation);
- solve multi-step linear equations (more than two steps, with one set of parentheses on each side of the equation);
- select the linear graphs that models the given real-world situation described in a narrative (no data set given);
- select the linear graph that models the given real-world situation described in a tabular set of data;
- evaluate an algebraic expression given values for one or more variables using grouping symbols and/or exponents less than four;
- determine the slope from the graph of a linear equation (no labeled points);
- apply the concept of rate of change to solve real-world problems;
- select the appropriate graphical representation of a given linear inequality;
- select the non-linear graph that models the given real-world situation or vice versa;
- identify the graphical representation of the solution to a one variable inequality on a number line.

At Level 3, the student is able to

- solve multi-step linear inequalities in real-world situations;
- recognize the graphical transformation that occurs when coefficients and/or constants of the corresponding linear equations are changed;
- determine the domain and/or range of a function represented by the graph of real-world situations.

Performance Indicators Teacher:

As documented through teacher observation –

At Level 1, the student is able to

- analyze rational number patterns (e.g., number of oranges in a rectangular pyramid display of 12 rows of oranges; row one has one orange, row two has four oranges, row three has nine oranges, etc.);
- describe in writing the pattern for real-world data listed in a function table (e.g., finance tables with various interest rates applied).

At Level 2, the student is able to

- produce an equation to describe the relationship between data sets (e.g., manufacturing, cost verses profit);
- solve a system of two linear equations using the graphing, elimination, and substitution methods, (e.g. manufacturing);
- defend the selection of a method for solving a system of equations (e.g., logical reasoning);

- represent algebraic expressions and operations using manipulative (e.g., drafting);
- model the steps for solving simple linear equations using manipulative (e.g., algebra tiles);
- write an equation that symbolically expresses a problem solving situation (e.g., robotics);
- justify correct results of algebraic procedures, (e.g., engineering, wind tunnel);
- distinguish between a function and other relationships (e.g., shipping, box dimensions vs. cost);
- solve quadratic functions using a variety of methods;
- analyze "families of functions" including non-linear functions (e.g., finance).

At Level 3, the student is able to

- analyze "families of functions" using technology (e.g., a technician is performing an experiment with a laser that is beamed at a mirror and checking its reflection);
- select the non-linear graph that models that models the given real-world situation described in a narrative (e.g., water patterns programmed for the musical productions at Opryland Hotel);
- explore patterns including Pascal's Triangle and a Fibonacci sequence (e.g., Forestry).

Sample Tasks: In a lab setting students gather data measuring the displacement of water as spheres are added to a 10 ml graduated cylinder. Students will graph this information and discover the connection of slope and the y-intercept to this set of data. Given various linear designs in a mock graphing calculator window, students duplicate the design applying principles of slope and y-intercepts in replicating the equation for each line of the design. Students design a quilt block on a coordinate grid system. Have students identify the equation for ten of the lines of the design stating the domain and range for that equation.

Linkages: Mathematics - Statistics and Probability. The use of patterns in other disciplines such as agribusiness, marketing, consumer science and industrial technology.

Standard 3.0: Geometry

The student will investigate, model, and apply geometric properties and relationships.

Learning Expectations:

The student will:

3.1 apply geometric properties, formulas, and relationships to solve real-world problems;
3.2 solve problems using the midpoint formula;
3.3 apply right triangle relationships including the Pythagorean Theorem and the distance formula;
3.4 find and represent solutions of quadratic equations geometrically.

Performance Indicators State:

As documented through state assessment –

At Level 1, the student is able to

- identify ordered pairs in the coordinate plane.

At Level 2, the student is able to

- apply the given Pythagorean Theorem to a real life problem illustrated by a diagram (no radicals in answer);

- apply proportion and the concepts of similar triangles to find the length of a missing side of a triangle.

At Level 3, the student is able to

- calculate the distance between two points given the Pythagorean Theorem and the distance formula.

Performance Indicators Teacher:

As documented through teacher observation –

At Level 1, the student is able to

- describe real-world uses of geometric formulas and relationships (e.g., construction);
- discuss issues related to estimating areas of irregular-shaped figures for real-world uses (e.g. fencing, painting, laying carpet, purchasing wallpaper or border);
- design a concept map showing connections among polygons (e.g. quadrilateral, parallelogram, rectangle, rhombus, square, and trapezoid).

At Level 2, the student is able to

- explain how to determine if a triangle is a right triangle given the measurements of all three sides (e.g. carpentry);
- illustrate the Pythagorean Theorem by measuring the length, width, and diagonals of rectangular objects. (e.g., surveying);
- design area models to illustrate the Pythagorean Theorem (e.g. construction);

At Level 3, the student is able to

- determine the height of an object that is difficult to measure by using the properties of similar triangles (e.g. electrical line technicians, which trees to trim);
- use a determinant to find the area of a right triangle graphed on a coordinate plane using appropriate technology (e.g. construction);
- explore relationships among varying dimensions in area and volume problems (e.g. gutter dimensions);
- apply the Pythagorean Theorem and the distance formula to workplace situations including appropriate approximations of irrational numbers (e.g., pluming);
- identify graphs of conic sections from their equations (e.g. space exploration).

Sample Task: Students read the coordinates of a right triangle on a map and calculate the area of the triangle using appropriate technology. While incorporating map scales, students check the reasonableness of their results by using the distance formula and the area formula.

Linkages: Mathematics - Estimation, Measurement, and Computation. Research and discuss the geometric applications in art. Research and write about how various geometric properties are used in careers such as construction, drafting, surveying, agribusiness, marketing, consumer science, and industrial technology.

Standard 4.0: Measurement

Students apply appropriate tools and units of measurement; develop effective estimation and computation strategies for producing reasonable results; and calculate using appropriate tools such as mental mathematics, technology, manipulatives, and pencil-and-paper.

Learning Expectations:

The student will:

4.1 use concepts of length, area, and volume to estimate and solve real-world problems;
4.2 apply and communicate measurement concepts and relationships in algebraic and geometric problem-solving situations;
4.3 demonstrate an understanding of rates and other derived and indirect measurements (, velocity, miles per hour, revolutions per minute, cost per unit);
4.4 make decisions about units, scales, and measurement tools that are appropriate for problem situations involving measurement;
4.5 analyze precision, accuracy, and approximate error in measurement situations.

Performance Indicators State:

As documented through state assessment –

At Level 1, the student is able to

- estimate the area of irregular geometric figures on a grid;
- calculate rates involving cost per unit to determine the best buy (no more than three samples);
- apply the given formula to determine the area or perimeter of a rectangle.

At Level 2, the student is able to

- apply the given formula to find the area of a circle, the circumference of a circle, or the volume of a rectangular solid.

At Level 3, the student is able to

- select the area representation for a given product of two one-variable binomials with positive constants and coefficients.

Performance Indicators Teacher:

As documented through teacher observation –

At Level 1, the student is able to

- justify the selection of a unit of measure in specific situations (e.g., manufacturing);
- refine strategies for estimating whole numbers, fractions, and percentages (e.g. cost);
- defend estimates of the perimeter and/or area of rectangles and triangles (e.g., flooring);
- discover and explain formulas used to compute area and volume (e.g., pool construction).

At Level 2, the student is able to

- describe the procedure for determining the area of a composite shape in a real-world situation (e.g., surveying);
- generalize area formulas using manipulatives for a parallelogram, a triangle, and a trapezoid (e.g. surveying);
- defend an estimate for the volume of a container (e.g. bottling companies);

- compare the height of a container to its volume and graph the relationship (e.g. packaging company);
- calculate a dimension of a geometric figure given the volume and other pertinent information (e.g., housing).

At Level 3, the student is able to

- discover the dimensions of a rectangle when given its area and the relationship between the length and width of the sides (e.g., art);
- describe how changes in the dimensions of figures affect perimeter, area, and volume (e.g., construction).

Sample Task: Use cubes to create models of differing sizes using a scaling factor. Determine the number of cubes representing the volume of each model. Then write an equation to show the volume of the nth figure (packaging industry).

Linkages: Mathematics – Geometry. Use formulas in Science. Discuss connections to drafting and carpentry, agribusiness, marketing, consumer science, and industrial technology.

Standard 5.0: Data Analysis and Probability

The student will collect, organize, represent, and interpret data and interpret and model situations to determine theoretical and experimental probabilities.

Learning Expectations:

The student will:

5.1 collect, represent, and describe linear and nonlinear data sets developed from the real world;
5.2 interpret a set of data using the appropriate measure of central tendency;
5.4 choose, construct, and analyze appropriate graphical representations for a data set;
5.5 understand the concept of random sampling;
5.6 apply counting principles of permutations and combinations using appropriate technology;
5.7 model situations to determine theoretical and experimental probabilities.

Performance Indicators State:

As documented through state assessment –

at Level 1, the student is able to

- determine the mean (average) of a given set of real-world data (no more than five two-digit numbers);
- interpret bar graphs representing real-world data;
- interpret circle graphs (pie charts) representing real-world data.

At Level 2, the student is able to

- choose the matching linear graph given a set of ordered pairs;
- make a prediction from the graph of a real-world linear data set;
- determine the median for a given set of real-world data (even number of data).

At Level 3, the student is able to

- apply counting principles of permutations or combinations in real-world situations.

Performance Indicators Teacher:

As documented through teacher observation –

At Level 1, the student is able to

- design a strategy for collecting real-world data for a scientific investigation (e.g., sampling); collect and organize real-world data (e.g., polling).

At Level 2, the student is able to

- graph real-world data using a variety of representations (e.g., newspaper);
- debate the selection of a graphical representation which best describes specific data (e.g., news media);
- model situations to determine theoretical and experimental probabilities (e.g., gaming);
- judge the validity of claims made in probabilistic situations (e.g., advertising);
- defend the sampling method chosen to conduct a survey (e.g., sales).

At Level 3, the student is able to

- debate possible conclusions that can be supported by the data (e.g., medical studies);
- make predictions from real-world data using a line of best fit (e.g., population studies);
- calculate standard deviation using the appropriate technology.

Sample Task: Students measure reaction time by dropping a meter stick between the thumb and fore finger of their partner. Repeat this measurement 3 times for each student. Record the cm measurement for the reaction time. Then calculate the mean, mode, median, and standard deviation of the generated data. Graph this data using several different types of graphs. Discuss the advantage of different graphical representations.

Linkages: Mathematics - Patterns, Functions, and Algebraic Thinking. Analyze census data. Research and discuss the careers that require the use of statistics such as statistician, actuaries, and scientist as well as technicians in agribusiness, marketing, consumer science and industrial vocations.

GEOMETRY

Course description:

Geometry uses problem situations, physical models, and appropriate technology to investigate and justify geometric concepts and relationships. Problem-solving situations provide all students an environment that promotes communication, engages student reasoning, and fosters connections within mathematics, to other disciplines, and to the real world. Students will use physical models to represent, explore, develop, and apply abstract concepts. The use of appropriate technology will help students develop mathematics needed in an increasingly technological world. The concepts and topics emphasized in the course include measurement, geometric patterns, coordinate geometry, two- and three-dimensional figures, transformational geometry, congruence, similarity, inductive and deductive reasoning, logic, and proof.

Standard Number 1.0: Number and Operations

Students will recognize, order, represent, and graph rational and irrational numbers, including absolute value notation.

Learning Expectations:

The student will:

1.1 demonstrate an understanding of the relative size of rational and irrational numbers;
1.2 choose and use appropriate notations for rational and irrational numbers, including graphic representations;
1.3 demonstrate an understanding of absolute value.

Performance Indicators State:

As documented through state assessment –

At Level 1, the student is able to

- order a set of rational and irrational numbers;
- find an integral power of a positive rational number (exponents 1-3).

At Level 2, the student is able to

- use absolute value to express the distance between two points on a number line and vice versa;
- simplify a radical (radicand less than 1000);
- match a given irrational number to the appropriate point on a number line and vice versa (e.g., √2, √30, pi).

There are no state-assessed performance indicators at Level 3.

Performance Indicators Teacher:

As documented through teacher observation –

At Level 1, the student is able to

- estimate the value of an irrational number expressed as a radical (radicand less than 1000);
- approximate pi given a table of values for the circumference and diameter of circles.

At Level 2, the student is able to

- use radicals and decimal approximations of irrational numbers to indicate calculated lengths or distances;
- discuss the relative accuracy of radicals and their decimal approximations.

At Level 3, the student is able to

- represent irrational numbers as lengths of lines in the coordinate plane (e.g. ,√5 is the length of the diagonal of a rectangle with base 1 and height 2).

Sample Task: Students will compute the hypotenuse of a given right triangles and arrange themselves in order from smallest to largest.

Linkages: Mathematics: Estimation, Measurement, and Algebra.

Standard 2.0: Algebra

Students will recognize, extend, create, and analyze a variety of geometric, spatial, and numerical patterns; solve real-world problems related to algebra and geometry; and use properties of various geometric figures to analyze and solve problems.

Learning Expectations:

The student will:

2.1 recognize, extend, and create geometric, spatial, and numerical patterns;
2.2 analyze mathematical patterns related to algebra and geometry in real-world problem solving;
2.3 solve problems connecting geometry with number theory, probability and statistics, and measurement and estimation using algebraic thinking and symbolism;
2.4 apply coordinate geometry to analyze and solve problems;
2.5 apply ratio and proportion to problems involving similar figures.

Performance Indicators State:

As documented through state assessment –

At Level 1, the student is able to

- extend or find missing element(s) in a geometric pattern;
- solve multistep linear equations to find length, width, perimeter, and area of geometric figures;
- apply the concept of rate of change to solve a real-world problem given a pattern of data;
- determine the slope given a graph of a linear equation and vice versa;
- determine the distance, midpoint, or slope when given the coordinates of two points (answers must be given as decimals to the nearest hundredth).

At Level 2, the student is able to

- determine the equation of a line parallel or perpendicular to a given line, from given information (e.g., equations of lines, graphs of lines, or two points);
- apply ratio and proportion to solve real-world problems involving polygons, (e.g., scale drawings, similar figures);
- apply the triangle inequality property to determine which sets of side lengths determine a triangle;
- determine the perimeter, area, or volume given the ratio of two similar polygons or rectangular solids;
- apply the Triangle Sum Theorem or Exterior Angle Theorem to determine the measures of the angles of a given triangle with the angle measures expressed algebraically.

At Level 3, the student is able to

- determine the equation of a circle given coordinates or the graph of the circle (e.g., the center, the endpoints of the diameter).

Performance Indicators Teacher:

As documented through teacher observation –

At Level 1, the student is able to

- apply the line of best fit given real-world data from geometric figures using technology (e.g., finding the interior angle sum of polygons when given the number of sides; find the circumference of circles when given the diameter).

At Level 2, the student is able to

- explore patterns in geometric situations (e.g., Fibonacci sequence and Golden Ratio);
- use manipulatives to determine relationships between linear, square, or cubic measures when one of the measures of the object has changed and represent algebraically.

At Level 3, the student is able to

- recognize complete and incomplete networks;
- graph plane figures on a coordinate plane and solve problems algebraically.

Sample Task: Students construct designs using basic geometric constructions. Then they transfer the design to a piece of 8""X 11""pane of plexiglass and paint the pane to create a "stained glass."

Linkages: Mosaic Tiling.

Standard 3.0: Geometry

Students will investigate, model, and apply geometric properties and relationships and use indirect reasoning to make conjectures; deductive reasoning to draw conclusions; and both inductive and deductive reasoning to establish the truth of statements.

Learning Expectations:

The student will:

3.1 analyze relationships among corresponding parts of similar or congruent geometric figures;
3.2 apply geometric properties of solids, polygons, and circles to solve real-world problems;
3.3 justify conclusions and solve problems using deductive reasoning;
3.4 use inductive reasoning to make conjectures and solve problems;
3.5 communicate position using spatial sense with two- and three-dimensional coordinate systems;
3.6 demonstrate an understanding of transformations of geometric figures (i.e., translations, rotations, dilations, and reflections);
3.7 apply right triangle relationships including the Pythagorean Theorem, the distance formula, and trigonometric ratios;
3.8 describe geometric objects and recognize minimal conditions necessary to define the geometric objects;
3.9 apply reflexive, transitive, and symmetric properties when appropriate;
3.10 demonstrate understanding of geometric properties of congruence, similarity, perpendicularity, and parallelism;
3.11 recognize and articulate relationships among families of geometric figures (e.g., quadrilaterals, prisms);
3.12 use logic and proof to establish the validity of conjectures and theorems.

Performance Indicators State:

As documented through state assessment:

At Level 1, the student is able to

- identify corresponding parts of similar and congruent geometric figures given a diagram.
- determine the length of a missing side in a right triangle when given two sides (answers must be given as simplified radicals).

At Level 2, the student is able to

- identify properties of plane figures from information given in a diagram;
- identify chords, inscribed angles, or central angles of circles given a diagram;
- determine congruence or similarity relations between triangles or quadrilaterals given a diagram;
- determine whether a plane figure has been translated, dilated, reflected, or rotated given a diagram and vice versa;
- solve problems involving complementary, supplementary, congruent, vertical, or adjacent angles given angle measures expressed algebraically;
- determine the trigonometric ratio for a right triangle needed to solve a real-world problem given a diagram;
- find a missing side length in a 30-60-90 or 45-45-90 degree triangle without rationalizing the denominator
- apply properties of quadrilaterals to solve a real-world problem given a diagram (opposite sides and angles, consecutive sides and angles, or diagonals);
- solve real-world problems involving measures of interior or exterior angles of regular polygons;
- identify the appropriate segment of a triangle given a diagram and vice versa (i.e. median, altitude, angle bisector, perpendicular bisector);
- determine which three-dimensional solid is represented by a given net and vice versa (two-dimensional drawing);
- determine the area of indicated regions involving circles, squares, rectangles, and/or triangles;
- justify triangle congruence given a diagram (i.e., ASA, SSS, AAS, SAS, or Hypotenuse/ Leg);

- determine if a triangle is a right triangle given the length of all the sides of a triangle.

At Level 3, the student is able to

- solve problems involving the properties of arcs, chords, tangents, or secants;
- find the area of a sector of a circle given a diagram.

Performance Indicators Teacher:

As documented through teacher observation –

At Level 1, the student is able to

- investigate the Pythagorean Theorem by constructing right triangles using a variety of manipulatives and/or technologies;
- construct parallelograms, rectangles, rhombi, and squares using physical materials, manipulatives, or technology.

At Level 2, the student is able to

- apply reflexive, transitive, or symmetric properties of equality or congruence;
- investigate the properties of angles, arcs, chords, tangents, and/or secants using technology or manipulatives;
- use inductive and deductive reasoning to make conjectures, draw conclusions, and solve problems;
- recognize and articulate relationships among families of geometric figures (e.g., quadrilaterals, prisms);
- write and defend indirect and direct proofs;
- use logical reasoning to solve problems in the real world;
- use manipulatives to explore the geometric mean of similar triangles;
- use appropriate tools or technology to develop geometric and spatial concepts;
- construct three-dimensional objects using physical materials and manipulatives;
- compare and construct quadrilateral properties using a variety of models (e.g., Venn diagrams, family trees, manipulative mobiles).

At Level 3, the student is able to

- use coordinates to communicate the location of a three-dimensional figure that has been rotated or reflected.

Sample Task: Students construct and use a hypsometer to measure several tall structures on the school grounds.

Linkages: Mathematics: Measurement. Surveying and Art.

Standard 4.0: Measurement

Students will apply appropriate units of measurement; develop effective estimation and computation strategies for solving real world problems involving length, area, and volume; and choose appropriate techniques and tools to measure quantities in order to meet specifications for precision, accuracy, and tolerance.

Learning Expectations:

The student will:

4.1 use concepts of length, area, and volume to estimate and solve real-world problems;
4.2 apply measurement concepts and relationships in algebraic and geometric problem-solving situations;
4.3 choose appropriate techniques and tools to measure quantities in order to meet specifications for precision, accuracy, and tolerance.

Performance Indicators State:

As documented through state assessment –

At Level 1, the student is able to

- determine the perimeter or area of a triangle or rectangle when the dimensions are given as first degree binomials in one variable;
- solve real world problems involving perimeter or area of three or four sided plane figures.

At Level 2, the student is able to

- determine the volume or surface area of a rectangular solid or cylinder in a real-world situation.

At Level 3, the student is able to

- determine whether a reading falls within an acceptable tolerance range.

Performance Indicators Teacher:

As documented through teacher observation –

At Level 1, the student is able to

- determine the measure of an angle using a protractor.

At Level 2, the student is able to

- construct bisectors of angles and line segments, perpendicular lines, congruent line segments and angles, and perpendicular bisectors using a variety of methods (e.g., patty paper, technology);
- solve problems involving volume of 3-dimensional figures, e.g. right prisms, pyramids, cones, cylinders, and spheres;
- solve problems involving surface area of prisms and cylinders.

At Level 3, the student is able to

- choose appropriate techniques and tools to measure quantities in order to meet specification for precision, accuracy, and tolerance;
- locate the irrational numbers $\sqrt{2}$ and $\sqrt{3}$ on a number line by using the Pythagorean relationship and a straightedge and compass, manipulatives, or technology;
- solve problems involving surface area of pyramids, cones, and spheres.

Sample Task: Students construct designs using basic geometric constructions. Then they transfer the design to a piece of 8" X 11" pane of plexiglass and paint the pane to create a "stained glass." Students construct one of the regular 3-dimensional solid and compute the volume and surface area.

Linkages: Mathematics – Geometry and Number & Operations. Surveying, construction, and architecture. .Mosaic Tiling.

Standard 5.0: Data Analysis and Probability

The student will investigate, explore, and apply geometric representations to calculate theoretical probability; and will use data from geometric figures to investigate relationships.

Learning Expectations:

The student will:

5.1 apply geometric representations to calculate theoretical probability;
5.2 use data analysis to investigate geometric relationships.

Performance Indicators State:

As documented through state assessment –

At Level 1, the student is able to

- make a prediction from a geometric representation of a real-world data set;

At Level 2, the student is able to

- determine the probability of an event represented as a subset of the area of a two-dimensional geometric figure.

There are no performance indicators for Level 3 of Data Analysis and Probability.

Performance Indicators Teacher:

As documented through teacher observation –

At Level 1, the student is able to

- explain and justify the given geometric representation of the probability of an event.

At Level 2, the student is able to

- use hands-on activities to model geometric representations of probability;
- collect and analyze data to make conjectures about geometric relationships.

At Level 3, the student is able to

- analyze and debate the validity of claims made based on the given theoretical probability of a real-world situation.

Sample Task: Construct two 1' X 1' dart boards and draw circular targets on each that are externally tangent to each adjacent circle and to the edge of the board. Draw two circles on one dartboard and three on the other. Throw randomly and count the throws that hit the board to determine which board yields the highest probability of a dart's landing in a circle. Calculate the probability for each bard.

Linkages: Game theory.

TECHNICAL GEOMETRY

Course description:

Technical Geometry incorporates the same core geometric concepts required in a standard geometry course but includes additional topics that focus on career and technical applications. These concepts will be taught using practical applications in a contextual style of teaching, including labs and projects. The structure of the course will include teaching groups of skills and concepts followed by their incorporation in a real world application and setting.

Standard 1.0: Number and Operations

Students will recognize, order, represent, and graph rational and irrational numbers.

Learning Expectations:

The student will:

1.1 demonstrate an understanding of the relative size of rational and irrational numbers;
1.2 choose and use appropriate notations for rational and irrational numbers, including graphic representations;
1.3 demonstrate an understanding of absolute value.

Performance Indicators State:

As documented through state assessment –

At Level 1, the student is able to

- order a set of rational and irrational numbers;
- find an integral power of a positive rational number (exponents 1-3).

At Level 2, the student is able to

- use absolute value to express the distance between two points on a number line and vice versa;
- simplify a radical (radicand less than 1000);
- match a given irrational number to the appropriate point on a number line and vice versa (e.g., √2, √30, pi).

There are no state-assessed performance indicators at Level 3.

Performance Indicators Teacher:

As documented through teacher observation –

At Level 1, the student is able to

- order a set of rational numbers (e.g., determine the sizing of electrical wire by gauge);
- estimate the value of an irrational number expressed as a radical (radicand less than 1000) (e.g., order lengths determined by a carpenter's square);
- approximate pi given a table of values for the circumference and diameter of circles.

At Level 2, the student is able to

- estimate the distance between two points on a line;
- discuss the accuracy of radicals and their decimal approximations in contexts such as carpentry.

At Level 3, the student is able to

- Represent irrational numbers as lengths of lines in the coordinate plane (e.g., $\sqrt{5}$ is the length of a diagonal brace of a rectangle frame with a base of 1 and a height of 2).

Sample Task: Students will compute the hypotenuse of given right triangles and arrange themselves in order from smallest to largest.

Linkages: Mathematics: Estimation, Measurement, and Algebra. Construction, auto mechanics, electrical, and plumbing.

Learning Expectations:

The student will:

2.1 recognize, extend, and create geometric, spatial, and numerical patterns;
2.2 analyze mathematical patterns related to algebra and geometry in real-world problem solving;
2.3 solve problems connecting geometry with number theory, probability and statistics, and measurement and estimation using algebraic thinking and symbolism;
2.4 apply coordinate geometry to analyze and solve problems;
2.5 apply ratio and proportion to problems involving similar figures.

Performance Indicators State:

As documented through state assessment –

At Level 1, the student is able to

- extend or find missing element(s) in a geometric pattern;
- solve multistep linear equations to find length, width, perimeter, and area of geometric figures;
- apply the concept of rate of change to solve a real-world problem given a pattern of data;
- determine the slope given a graph of a linear equation and vice versa;
- determine the distance, midpoint, or slope when given the coordinates of two points (answers must be given as decimals to the nearest hundredth).

At Level 2, the student is able to

- determine the equation of a line parallel or perpendicular to a given line, from given information (e.g., equations of lines, graphs of lines, or two points);
- apply ratio and proportion to solve real-world problems involving polygons, (e.g., scale drawings, similar figures);
- apply the triangle inequality property to determine which sets of side lengths determine a triangle;
- determine the perimeter, area, or volume given the ratio of two similar polygons or rectangular solids;

- apply the Triangle Sum Theorem or Exterior Angle Theorem to determine the measures of the angles of a given triangle with the angle measures expressed algebraically.

At Level 3, the student is able to

- determine the equation of a circle given coordinates or the graph of the circle (e.g., the center, the endpoints of the diameter).

Performance Indicators Teacher:

As documented through teacher observation –

At Level 1, the student is able to

- apply the line of best fit given real-world data to make predictions and describe trends (e.g., quality control sampling, marketing sales of specific products, demographics in an area);
- translate given data into algebraic expressions (e.g., feasibility studies, cost vs. profit/production, and consumer costs for products);
- find powers and roots of numbers in problem solving using appropriate technology (e.g., apply concept to bacterial growth, amortization of a house).

At Level 2, the student is able to

- explore patterns in real-world situations (e.g., Golden Ratio, Pythagorean Triples, and Tiling);
- use manipulatives to determine relationships between linear, square, or cubic measures when one of the measures of the object has changed;
- simplify radicals to estimate irregular areas (e.g., building design, building braces, and grade of a slope);
- find regression equations for data sets using technology;
- solve problems involving indirect ratios, such as gear ratios.

At Level 3, the student is able to

- recognize complete and incomplete networks (e.g., delivery routes, mapping, electrical and plumbing applications);
- apply the Law of Sines and Law of Cosines to triangles (e.g., surveying, architecture, plotting location, welding).

Standard 3.0: Geometry

Students will investigate, model, and apply geometric properties and relationships and use indirect reasoning to make conjectures; deductive reasoning to draw conclusions; and both inductive and deductive reasoning to establish the truth of statements.

Learning Expectations:

The student will:

3.1 analyze relationships among corresponding parts of similar or congruent geometric figures;
3.2 apply geometric properties of solids, polygons, and circles to solve real-world problems;
3.3 justify conclusions and solve problems using deductive reasoning;
3.4 use inductive reasoning to make conjectures and solve problems;
3.5 communicate position using spatial sense with two- and three-dimensional coordinate systems;

3.6 demonstrate an understanding of transformations of geometric figures (i.e., translations, rotations, dilations, and reflections);
3.7 apply right triangle relationships including the Pythagorean Theorem, the distance formula, and trigonometric ratios;
3.8 describe geometric objects and recognize minimal conditions necessary to define the geometric objects;
3.9 apply reflexive, transitive, and symmetric properties when appropriate;
3.10 demonstrate understanding of geometric properties of congruence, similarity, perpendicularity, and parallelism;
3.11 recognize and articulate relationships among families of geometric figures (e.g., quadrilaterals, prisms);
3.12 use logic and proof to establish the validity of conjectures and theorems.

Performance Indicators State:

As documented through state assessment:

At Level 1, the student is able to

- identify corresponding parts of similar and congruent geometric figures given a diagram.
- determine the length of a missing side in a right triangle when given two sides (answers must be given as simplified radicals).

At Level 2, the student is able to

- identify properties of plane figures from information given in a diagram;
- identify chords, inscribed angles, or central angles of circles given a diagram;
- determine congruence or similarity relations between triangles or quadrilaterals given a diagram;
- determine whether a plane figure has been translated, dilated, reflected, or rotated given a diagram and vice versa;
- solve problems involving complementary, supplementary, congruent, vertical, or adjacent angles given angle measures expressed algebraically;
- determine the trigonometric ratio for a right triangle needed to solve a real-world problem given a diagram;
- find a missing side length in a 30-60-90 or 45-45-90 degree triangle without rationalizing the denominator
- apply properties of quadrilaterals to solve a real-world problem given a diagram (opposite sides and angles, consecutive sides and angles, or diagonals);
- solve real-world problems involving measures of interior or exterior angles of regular polygons;
- identify the appropriate segment of a triangle given a diagram and vice versa (i.e. median, altitude, angle bisector, perpendicular bisector);
- determine which three-dimensional solid is represented by a given net and vice versa (two-dimensional drawing);
- determine the area of indicated regions involving circles, squares, rectangles, and/or triangles;
- justify triangle congruence given a diagram (i.e., ASA, SSS, AAS, SAS, or Hypotenuse/ Leg);
- determine if a triangle is a right triangle given the length of all the sides of a triangle.

At Level 3, the student is able to

- solve problems involving the properties of arcs, chords, tangents, or secants;
- find the area of a sector of a circle given a diagram.

Performance Indicators Teacher:

As documented through teacher observation –

At Level 1, the student is able to

- use indirect reasoning to make conjectures and solve problems (e.g., crop analysis, inventory);
- investigate the Pythagorean Theorem using various technologies;
- construct parallelograms, rectangles, rhombi, and squares using physical materials, manipulatives, or technology (e.g., building construction, automobile design, quilting, pattern design).

At Level 2, the student is able to

- use inductive and deductive reasoning to draw a conclusion (e.g., diagnostics in automotives, health science nutrition science);
- recognize and articulate relationships among families of geometric figures (e.g., metal fabrication, floral design, landscaping);
- investigate the properties of angles, arcs, chords, tangents, and/or secants using technology or manipulatives;
- use logical reasoning to solve problems in the real world (e.g., health science, agri-science, criminal justice, nutrition);
- use manipulatives to explore the geometric mean of similar triangles (e.g., plumbing, electrical wiring);
- use appropriate technology to develop geometric and spatial concepts;
- construct three-dimensional objects using physical materials and manipulatives (e.g., packaging, cake decorating, building construction, sculptural design, and mobile creation in child care);
- identify the three basic trig ratios and their graphs;
- recognize and apply reflexive, symmetric, and transitive properties of equality, similarity, and congruence.

At Level 3, the student is able to

- use coordinates to communicate the location of a three-dimensional figure that has been rotated or reflected (e.g., systems, diagnostics, CAD);
- apply the three basic trig ratios to solving problems (e.g., angle of elevation, grade of a road, bearings).

Sample Task: Students construct and use a hypsometer to measure several tall structures on the school grounds.

Linkages: Mathematics: Measurement. Surveying, Design, Road Construction, and Art.

Standard 4.0: Measurement

Students will apply appropriate units of measurement; develop effective estimation and computation strategies for solving real world problems involving length, area, and volume; and choose appropriate techniques and tools to measure quantities in order to meet specifications for precision, accuracy, and tolerance.

Learning Expectations:

The student will:

4.1 use concepts of length, area, and volume to estimate and solve real-world problems;
4.2 apply measurement concepts and relationships in algebraic and geometric problem-solving situations;
4.3 choose appropriate techniques and tools to measure quantities in order to meet specifications for precision, accuracy, and tolerance.

Performance Indicators State:

As documented through state assessment –

At Level 1, the student is able to

- determine the perimeter or area of a triangle or rectangle when the dimensions are given as first degree binomials in one variable;
- solve real world problems involving perimeter or area of three or four sided plane figures.

At Level 2, the student is able to

- determine the volume or surface area of a rectangular solid or cylinder in a real-world situation.

At Level 3, the student is able to

- determine whether a reading falls within an acceptable tolerance range.

Performance Indicators Teacher:

As documented through teacher observation –

At Level 1, the student is able to

- determine the measure of an angle using a protractor (angles of elevation and grade of a road in surveying);
- recognize vector quantities;
- determine the best estimate for a given measurement.

At Level 2, the student is able to

- construct bisectors of angles and line segments, perpendicular lines, congruent line segments and angles, and perpendicular bisectors using a variety of methods such as patty paper and technology (e.g., tailoring of clothes, dentistry, carpentry); draw auxiliary diagrams to help solve for an unknown dimension or an unknown angle;
- choose appropriate techniques and tools to measure quantities in order to meet specification for precision, accuracy, and tolerance (quality control assurance, jewelry, tool and die);
- solve problems involving volume of three dimensional figures, e.g., right prisms, pyramids, cones, cylinders and spheres;
- solve problems involving surface area of prisms and cylinders;
- make simple scale drawings (e.g., blueprints, models, publishing);
- find the magnitude and direction of a vector (e.g., location, headings).

At Level 3, the student is able to

- locate the irrational numbers √2 and √3 on a number line by using the Pythagorean relationship and a straightedge and compass (e.g., surveying);
- solve problems involving surface area of pyramids, cones and spheres;
- solve problems involving signed numbers and vectors (e.g., work, force, bearings).

Sample Task: Students construct designs using basic geometric constructions. Then they transfer the design to a piece of 8""X 11""pane of plexiglass and paint the pane to create a "stained glass." Students construct one of the regular 3-dimensional solid and compute the volume and surface area.

Linkages: Mathematics – Geometry and Number & Operations. Surveying, construction, and architecture. .Mosaic Tiling.

Standard 5.0: Data Analysis and Probability

The student will investigate, explore, and apply geometric representations to calculate theoretical probability; and will use data from geometric figures to investigate relationships.

Learning Expectations:

The student will:

- apply geometric representations to calculate theoretical probability;
- use data analysis to investigate geometric relationships.

Performance Indicators State:

As documented through state assessment –

At Level 1, the student is able to

- make a prediction from a geometric representation of a real-world data set;

At Level 2, the student is able to

- determine the probability of an event represented as a subset of the area of a two-dimensional geometric figure.

There are no performance indicators for Level 3 of Data Analysis and Probability.

Performance Indicators Teacher:

As documented through teacher observation –

At Level 1, the student is able to

- explain and justify the given geometric representation of the probability of an event (sales projections).

At Level 2, the student is able to

- use hands-on activities to model geometric representations of probability (polling results, inventory control).

At Level 3, the student is able to

- analyze and debate the validity of claims made based on the given theoretical probability of a real-world situation (defective parts in a sample of products, analysis of the validity of survey results).

Algebra II

Course Description:

Algebra II is a course that uses problem situations, physical models, and appropriate technology to extend algebraic thinking and engage student reasoning. Problem-solving situations will provide all students an environment that promotes communication and fosters connections within mathematics, to other disciplines, and to the real world. Students will use physical models to represent, explore, and develop abstract concepts. The use of appropriate technology will help students apply mathematics in an increasingly technological world. The concepts emphasized in the course include analysis of "family of functions," solving systems of equations, graphing, data analysis, and logarithmic and exponential functions.

Standard Number 1.0: Number and Operations

Students will recognize, represent, model, and apply real numbers and operations and will demonstrate an understanding of properties and operations of the complex number system.

Learning Expectations:

The student will:

1.1 demonstrate an understanding of the subsets, elements, properties, and operations of the complex number system;
1.2 connect physical, graphical, verbal, and symbolic representations of real numbers;
1.3 articulate, model, and apply the concept of inverse (e.g. opposites, reciprocals, and powers and roots);
1.4 describe, model, and apply inverse operations;
1.5 connect physical, graphical, verbal, and symbolic representations of absolute value;
1.6 use a variety of notations appropriately (e.g. logarithmic, factorial, sigma, delta, radical);
1.7 perform operations on algebraic expressions and informally justify the procedures chosen.

Performance Indicators State:

As documented through state assessment –

At Level 1, the student is able to

- order a given set of real numbers;
- identify the reciprocal of a real number;
- multiply two polynomials with each factor having no more than two terms.

At Level 2, the student is able to

- perform basic operations using complex numbers (i.e., addition, subtraction, and multiplication);
- select a graph that represents an absolute value equation on a coordinate plane;
- identify the exponential form of a logarithmic expression and vice versa;
- simplify expressions with rational and negative exponents;
- add, subtract, and multiply algebraic expressions.

At Level 3, the student is able to

- determine the conjugate of a complex number.

Performance Indicators Teacher:

As documented through teacher assessment –

At Level 1, the student is able to

- probe the relationships among various subsets of the real-number system;
- explore various representations of absolute value on a number line;
- use ratios and proportions to represent real-world problems;
- use estimation to determine a reasonable solution for a tedious arithmetic computation of a real-world situation that may involve unit conversions;
- investigate product and factoring patterns of polynomials;
- compare and contrast the GCF and the LCM of a set of algebraic expressions;
- add, subtract, and perform scalar multiplication on matrices using appropriate technology.

At Level 2, the student is able to

- analyze the relationships among sets of numbers using a Venn diagram of the complex number system;
- use delta notation to represent the rate of change in a real-world situation;
- use the inverse notation of powers and roots;
- perform basic operations on rational algebraic expressions.

At Level 3, the student is able to

- justify the procedures chosen when performing operations on algebraic expressions and equations;
- use factorial notation for coefficients in a binomial expansion;
- determine the multiplicative inverse of a complex number;
- formulate the representation of a series using sigma notation.

Sample Task: Students design and build a simple fractal from available materials.

Linkages: Mathematics – Estimation, Measurement, and Computation. Make connections to concept mapping in literature, language arts, and social studies. Connect estimation and computation strategies to business and finance.

Standard 2.0: Algebra

Students will describe, extend, analyze, and create a wide variety of patterns and functions using appropriate materials and representations in real-world problem solving, and will demonstrate an understanding of the behavior of a variety of functions and their graphs.

Learning Expectations:

The student will:

2.1 analyze mathematical patterns related to algebra and geometry in real-world problem solving;
2.2 use algebraic thinking to generalize a pattern by expressing the pattern in functional notation;
2.3 solve linear systems using a variety of techniques, including matrices;
2.4 communicate the meaning of variables in algebraic expressions, equations, and inequalities;
2.5 manipulate the algebraic functions with constants and analyze graphs to describe the behavior of functions;
2.6 apply the concept of rate of change;
2.7 identify and represent a variety of functions (e.g. linear, quadratic, cubic);
2.8 identify, describe, and articulate the characteristics and the parameters of a parent function;
2.9 interpret results of algebraic procedures;
2.10 apply the concept of variable in simplifying algebraic expressions, solving equations, and solving inequalities;
2.11 interpret graphs that depict real-world phenomena;
2.12 model real-world phenomena using functions and graphs;
2.13 describe the domain and range of functions and articulate restrictions imposed either by the operations or by the real-life situations which the functions represent;
2.14 use linear programming to solve real-world problems.

Performance Indicators State:

As documented through state assessment –

At Level 1, the student is able to

- translate a verbal sentence into an algebraic equation and vice versa;
- select the algebraic equation that generalizes the pattern represented by data in a given table;
- solve multi-step (more than two steps) linear equations (one set of parentheses on each side of the equations and/or variables on both sides);
- select the graph that represents a given linear function expressed in slope-intercept form;
- select the graph that models a given real-world situation (i.e., linear and non-linear);
- identify the graphical representation of the solution to a one-variable inequality on a number line.

Level 2, the student is able to

- select functional notation to generalize a given numeric pattern;
- solve one-variable linear equations with rational expressions;
- select the graph of a two-variable inequality;
- determine the domain of polynomial, rational, square root, exponential and logarithmic functions;
- determine the range of a wide variety of functions given a graph;
- solve a system of linear equations with 2 variables (e.g. substitution, elimination, Cramer's Rule, and graphing);
- apply properties of logarithms to simplify a logarithmic expression;
- identify matrices that model given real-world situations.

At Level 3, the student is able to

- determine the inverse of a logarithmic function given its graph.

Performance Indicators Teacher:

As documented through teacher observation –

At Level 1, the student is able to

- explain what the changes in slope of a non-linear graph representing a real-world situation;
- analyze mathematical patterns related to algebra and geometry in real-world problem solving.

At Level 2, the student is able to

- collect real-world data to make generalizations;
- apply results of algebraic procedures to real-world situations;
- use a variety of methods to solve linear systems in two and three variables (e.g., elimination, substitution, Cramer's Rule, matrices, and graphing);
- explain the restrictions on the variable in a radical equation;
- choose an appropriate method to find the roots of a quadratic equation (e.g. completing the square, quadratic formula, factoring, or graphing calculator);
- solve quadratic inequalities;
- construct matrices given real world situation.

At Level 3, the student is able to

- evaluate the graph of a function to determine if it is periodic;
- sketch a system of linear inequalities and determine the maximum or minimum value of the related function;
- justify the procedures chosen when performing operations on algebraic expressions and equations;
- find the maximum or minimum value given the graph of the feasible region of the real world linear programming application;
- determine all the roots of a higher order polynomial (i.e., Descartes' Rule of Signs, Rational Root Theorem, and Synthetic Division).

Sample Task: Examine patterns found in Pascal's Triangle.

Linkages: Mathematics: Statistics and Probability. Data analysis and pattern recognition in science.

Standard 3.0: Geometry

Students will investigate, model, and apply geometric properties and relationships.

Learning Expectations:

The student will:

3.1 apply geometric properties, formulas, and relationships to solve real-world problems;
3.2 justify conclusions using deductive reasoning;
3.3 use inductive reasoning to make conjectures;
3.4 communicate position using spatial sense with two- and three-dimensional coordinate systems;
3.5 perform a given transformation and predict the results of the transformation.

Performance Indicators State:

As documented through state assessment –

At Level 1, the student is able to

- apply the given Pythagorean Theorem to real-world problems.

At Level 2, the student is able to

- predict the graphical transformation that occurs when coefficients and/or constants of given function are changed (no trigonometric or logarithmic functions);
- apply proportion and the concepts of similar triangles to solve real world problems.

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At Level 3, the student is able to

- describe the transformation that has changed a "parent function" to the given related function (e.g., right shift of 3 units, reflection in the x-axis).

Performance Indicators Teacher:

As documented through teacher observation –

At Level 1, the student is able to

- estimate the irrational solution of a real-world problem using the Pythagorean Theorem.

At Level 2, the student is able to

- apply the distance formula to obtain the equation of a circle in order to solve real-world problems;
- use deductive reasoning to draw conclusions.

At Level 3, the student is able to

- use matrices to find the area of a triangle on a coordinate plane;
- investigate and explore the conics section.

Sample Task: Students use properties of similar triangles to determine the height of objects that are difficult to measure.

Linkages: Research and discuss geometric applications such as art.

Standard 4.0: Measurement

The student will understand and be able to apply the units, systems and processes of measurement.

Learning Expectations:

The student will:

4.1 apply measurement concepts and relationships in algebraic and geometric problem-solving situations;
4.2 apply appropriate techniques, tools, and formulas to determine measurements.

Performance Indicators State:

As documented through state assessment –

At Level 1, the student is able to

- select the area representation for a given product of two binomials.

At Level 2, the student is able to

- apply the given formula to find area and circumference of circles, area and perimeter of polygons, and volume of regular solids;

At Level 3, the student is able to

- solve real world problems given logarithmic and exponential formulas (e.g. Ph scale, Richter scale.).

Performance Indicators Teacher:

As documented through teacher observation –

At Level 1, the student is able to

- select the appropriate unit of measure given the real world situation.

At Level 2, the student is able to

- use appropriate measurements in collecting data for a real world situation.

There are no teacher performance indicators at *Level 3.*

Sample Task: Students construct designs using basic geometric constructions. Then they transfer the design to a piece of 8" X 11" pane of plexiglass and paint the pane to create a "stained glass." Students construct one of the regular 3-dimensional solid and compute the volume and surface area.

Linkages: Mathematics – Geometry and Number & Operations. Surveying, construction, and architecture. .Mosaic Tiling.

Standard 5.0: Data Analysis and Probability

The student will collect, organize, represent, and interpret data; make and evaluate inferences and predictions; present and evaluate arguments based on data analysis; and model situations to determine theoretical and experimental probabilities.

Learning Expectations:

The student will:

5.1 understand concept of randomness in sampling;

5.2 apply appropriate technology in data collection and analysis;
5.3 apply counting principles of permutations and combinations using appropriate technology;
5.4 apply theoretical and experimental probability to analyze the likelihood of an event;
5.5 collect, represent, and describe linear and nonlinear data sets developed from real world;
5.6 make predictions from a data set using curve fitting with appropriate technology;
5.7 make inferences about a data set using appropriate measures of central tendency and dispersion;
5.8 describe and apply the normal distribution and its properties;
5.9 identify mutually exclusive and non-mutually exclusive events;
5.10 analyze the probability of dependent events and of independent events;
5.11 use simulations to estimate probability;
5.12 choose, construct, and analyze appropriate graphical representations for a data set;
5.13 analyze the validity of statistical conclusions and the use, misuse, and abuse of data.

Performance Indicators State:

As documented through state assessment –

At Level 1, the student is able to

- make a prediction from the graph of a real-world data set;
- determine the measures of central tendency for a given set of real-world data;
- choose the matching linear graph when given a set of ordered pairs representing real-world data.

At Level 2, the student is able to

- categorize the correlation of a scatterplot using real-world data (i.e., positive, negative, strong, or weak);
- determine the number of possible outcomes for a given experiment (i.e. the multiplication counting principle, permutations, or combinations);
- determine the theoretical probability of a simple event for a given situation;
- determine the theoretical probability of a compound event (i.e., dependent or independent, union and intersection).

At Level 3, the student is able to

- find the equation for the line of best fit given a scatterplot depicting real-world data.

Performance Indicators Teacher:

As documented through teacher observation –

At Level 1, the student is able to

- analyze student-collected data to make predications or generalizations.

At Level 2, the student is able to

- use simulations to help predict the probability of a given situations;
- determine the theoretical probability of mutually exclusive events for a given situation;
- analyze theoretical or experimental probability to determine the likelihood of an event;

- analyze data using linear and quadratic functions using the appropriate technology;
- analyze the validity of statistical conclusions and the use, misuse, and abuse of data;
- identify the mean and the standard deviation given the graph of a normal distribution.

At Level 3, the student is able to

- use the measure of central tendency which best represents the given real-world data set given a distribution curve.

Sample Task: Students analyze real-world data collected from the newspaper and explore and report the uses, misuses, and abuses of reported statistical data. Students search the internet to collect age and market value of a selected vehicle over a specific period of time. They use a graphing calculator to create a scatterplot and construct a line of best fit to predict the depreciation of the vehicle.

Linkages: Sports; social studies; economics.

Integrated Mathematics I

Course Description:

This course is the first of three courses in a series that uses a more integrated approach to cover the same algebra and geometry concepts and skills that are included in the traditional three course series. The problem situations, models, and technology used will foster connections among the various strands of mathematics and develop concepts from multiple perspectives.

Standard 1.0: Number and Operations

Students will recognize, represent, model, and apply real numbers and operations verbally, physically, symbolically, and graphically.

Learning Expectations:

The student will:

1.1 demonstrate an understanding of the elements, subsets, properties, and operations of rational numbers;
1.2 demonstrate understanding of positive integer exponents and perform operations with expressions involving exponents;
1.3 connect physical, graphical, verbal, and symbolic representations of rational numbers;
1.4 connect physical, graphical, verbal, and symbolic representations of absolute value;
1.5 articulate, model, and apply the concept of inverse (i.e. opposites and reciprocals);
1.6 describe, model, and apply inverse operations;
1.7 perform operations on algebraic expressions and informally justify the procedures chosen;
1.8 apply matrix addition, subtraction, and scalar multiplication in real- world problems (e.g. inventory), using appropriate technology;
1.9 use a variety of notations appropriately (e.g., exponential, functional, square root);
1.10 select and apply an appropriate method (i.e. mental arithmetic, paper and pencil, or technology) for computing with real numbers, and use estimation to evaluate the reasonableness of the result.

Student Performance Indicators:

At Level 1, the student is able to

- select the best estimate for the coordinate of a given point on a number line (only rational);
- identify the opposite of a rational number;
- determine the square root of a perfect square less than 169;
- use exponents to simplify a monomial written in expanded form;
- apply order of operations when computing with integers using no more than two sets of grouping symbols and exponents 1 and 2;
- select a reasonable solution for a real-world division problem in which the remainder must be considered;
- compare and contrast the GCF and LCM of a set of numbers.

At Level 2, the student is able to

- probe the relationships among various subsets of the real number system;
- compare and contrast the GCF and LCM of a set of algebraic expressions;
- order a given set of rational numbers (both fraction and decimal notations);

- identify the reciprocal of a rational number;
- add and subtract algebraic expressions;
- multiply two polynomials with each factor having no more than two terms;
- use estimation to determine a reasonable solution for a tedious arithmetic computation;
- select ratios and proportions to represent real-world problems (e.g. scale drawings, sampling);
- perform operations on matrices using appropriate technology (addition, subtraction, and scalar multiplication).

At Level 3, the student is able to

- apply the concept of slope to represent rate of change in a real-world situation;
- scrutinize approximate values of real numbers such as pi and the square root of two.

Sample Task: Students research the history of prime numbers and their uses.

Linkages: Make connections to types of numbers used in science, social studies, and finance.

Standard 2.0: Algebra

Students will describe, extend, analyze, and create a wide variety of patterns and functions using appropriate materials and representations in real world problem solving.

Learning Expectations:

The student will:

2.1 communicate the meaning of variables in algebraic expressions, equations, and inequalities;
2.2 identify dependent and independent variables in real-world situations;
2.3 apply the concept of variable in simplifying algebraic expressions, solving equations, and solving inequalities;
2.4 represent the solution set linear equations and inequalities in one variable symbolically, graphically, and verbally;
2.5 interpret graphs that depict real-world phenomena;
2.6 model real-world phenomena using graphs;
2.7 represent functions with equations, graphs, tables, and words;
2.8 understand and apply slope as rate of change;
2.9 solve real-world problems represented by linear functions and interpret the slope and intercepts;
2.10 solve systems of two equations in two unknowns using a variety of techniques;
2.11 recognize and extend numerical, geometric, and spatial patterns;
2.12 describe the domain and range of functions imposed either by operations or by real-life situations that the functions represent;
2.13 describe the transformation of the graph that occurs when coefficients and/or constants of the corresponding linear equation are changed;
2.14 generalize numerical, geometric patterns verbally and symbolically.

Student Performance Indicators:

At Level 1, the student is able to

- extend a geometric pattern;
- extend a numerical pattern;
- translate a verbal expression into an algebraic expression;
- evaluate a first degree algebraic expression given values for one or more variables;

- solve one- and two-step linear equations using integers (with integral coefficients and constants).

At Level 2, the student is able to

- select the algebraic notation which generalizes the pattern represented by data in a given table;
- translate a verbal sentence into an algebraic equation;
- select the graph that represents a given linear function expressed in slope-intercept form;
- solve multi-step linear equations (more than two steps, variables on only one side of the equation);
- solve multi-step linear equations (more than two steps, with variables on both sides of the equation);
- solve multi-step linear equations (more than two steps, with one set of parentheses on each side of the equation);
- select the linear graph that models the given real-world situation described in a narrative (no data set given);
- select the linear graph that models the given real-world situation described in a tabular set of data;
- evaluate an algebraic expression given values for one or more variables using grouping symbols and/or exponents less than four;
- determine the slope (rate of change) from the graph of a linear equation (no labeled points);
- apply the concept of rate of change to solve real-world problems;
- select the appropriate graphical representation of a given linear inequality;
- select the non-linear graph that models the given real-world situation or vice versa;
- identify the graphical representation of the solution to a one variable inequality on a number line;
- produce an equation to describe the relationship between data sets;
- explore patterns including Pascal's Triangle and a Fibonacci sequence;
- solve a system of two linear equations using the graphing, elimination, and substitution methods;
- defend the selection of a method for solving a system of equations;
- represent algebraic expressions and operations using manipulatives;
- model the steps for solving simple linear equations using manipulatives;
- write an equation that symbolically expresses a problem solving situation;
- justify correct results of algebraic procedures;
- distinguish between a function and other relationships.

At Level 3, the student is able to

- solve multi-step linear inequalities in real-world situations;
- analyze "families of functions" using technology;
- determine the domain and/or range of a function represented by the graph of real-world situations;
- select the system of equations that could be used to solve a given real-world problem;
- find the solution to a quadratic equation given in standard form (integral solutions and a leading coefficient of one).

Sample Task: Students use an almanac or the internet to find the area and the average depth of the world's ten largest bodies of salt water. Then they draw a scatterplot showing the relationship between these two sets of data, and describe the relationship and determine if it is a functional relationship.

Linkages: Mathematics – Probability and Statistics. Patterns in other disciplines and in a variety of cultures.

Standard 3.0: Geometry

Students will investigate, model, and apply geometric properties and relationships.

Learning Expectations:

The student will:

3.1 apply inductive reasoning in making conjectures, then test conjectures and/or determine a counterexample;
3.2 apply properties of special pairs of angles (e.g. supplementary, complementary, vertical, and adjacent);
3.3 articulate relationships of angles formed when parallel lines are cut by a transversal;
3.4 apply the concept of slope to parallel and perpendicular lines;
3.5 solve real world problems involving length, perimeter, and circumference;
3.6 apply the properties of congruence and similarity to solve problems;
3.7 apply the Pythagorean Theorem and the distance formula;
3.8 use appropriate measurement techniques and tools in investigating properties of polygons (triangle angle properties, angles of polygons, and triangle inequalities).

Student Performance Indicators:

At Level 1, the student is able to

- describe real-world uses of geometric formulas and relationships;
- discuss issues related to estimating areas of irregular-shaped figures for real-world uses (i.e. fencing, painting, laying carpet, purchasing wallpaper or border);
- identify ordered pairs in the coordinate plane.

At Level 2, the student is able to

- apply the given Pythagorean Theorem to a real life problem illustrated by a diagram (no radicals in answer);
- apply proportion and the concepts of similar triangles to find the length of a missing side of a triangle.

At Level 3, the student is able to

- calculate the distance between two points given the Pythagorean Theorem and the distance formula;
- determine the height of an object that is difficult to measure by using the properties of similar triangles.

Sample Task: Approximate the value of pi (π) by looking at the relationship between the diameter and circumference of various circular objects after measuring using a string or a tape measure. Students research and write about how various geometric properties are used in careers such as construction, drafting, and surveying.

Linkages: Mathematics - Estimation, Measurement, and Computation, Research, and the geometric applications in art.

Standard 4.0: Measurement

The student will choose appropriate techniques and tools to measure quantities in order to meet specifications for precision and accuracy;

Learning Expectations:

The student will

4.1 choose appropriate techniques and tools to measure quantities in order to meet specifications for precision and accuracy;
4.2 use concepts of length, area, and volume to estimate and solve real- world problems;
4.3 apply measurement concepts, relationships, and formulas in algebraic and geometric problem-solving situations;
4.4 use estimation to make predictions and determine reasonableness of results;
4.5 demonstrate an understanding of rates and other derived and indirect measurements (e.g. velocity, miles per hour, revolutions per second, and cost per unit).

Student Performance Indicators:

At Level 1, the student is able to

- estimate the area of irregular geometric figures on a grid;
- calculate rates involving cost per unit to determine the best buy (no more than three samples)'
- apply the given formula to determine the area or perimeter of a rectangle.

At Level 2, the student is able to

- apply the given formula to find the area of a circle, the circumference of a circle, or the volume of a rectangular solid;
- defend estimates of the perimeter and/or area of rectangles and triangles.

At Level 3, the student is able to

- select the area representation for a given product of two one-variable binomials with positive constants and coefficients.

- describe how changes in the dimensions of figures affect perimeter, area, and volume.

Sample Task: Place students in small groups giving each group a different length of string. Have each group form a rectangle with the string. Ask each group to measure the sides of their rectangle and find its area. Using the string, direct each group to construct the rectangle with the greatest possible area. Give each group the opportunity to justify their solution.

Linkages: Mathematics – Geometry. Use formulas in Science. Discuss connections to drafting and carpentry. Connect estimation and computation strategies to business and finance.

Standard 5.0: Data Analysis and Probability

Learning Expectations:

The student will

5.1 collect, represent, and describe linear and nonlinear data sets developed from the real world using appropriate technology;

5.2 choose, construct, and analyze appropriate graphical representations for a data set;
5.3 interpret data using the appropriate measure of central tendency for the data set;
5.4 determine the measures of dispersion of a data set including range and quartiles;
5.5 apply basic counting principles, introducing factorial notation; apply experimental and theoretical probability with simulations where appropriate;
5.6 make predictions from a linear data set using a line of best fit.

Student Performance Indicators:

At Level 1, the student is able to

- determine the mean (average) of a given set of real-world data (no more than five two-digit numbers);
- interpret bar graphs representing real-world data;
- interpret circle graphs (pie charts) representing real-world data.

At Level 2, the student is able to

- graph real-world data using a variety of representations;
- choose the matching linear graph given a set of ordered pairs;
- make a prediction from the graph of a real-world linear data set;
- determine the median for a given set of real-world data (even number of data).

At Level 3, the student is able to

- apply counting principles of permutations or combinations in real-world situations;
- debate possible conclusions that can be supported by the data;
- make predictions from real-world data using a line of best fit.

Sample Task: Students research the age of each Tennessee governor at the time of his/her inauguration. The students organize their information and will determine which measure of central tendency is the best description of the data. Students explain their decision.

Linkages: Mathematics - Patterns, Functions, and Algebraic Thinking. Analyze census data. Research and discuss the careers that require the use of statistics such as statistician, actuaries, and scientists.

Integrated Mathematics II

Course Description:

This course is the second of three courses in a series that uses a more integrated approach to cover the same algebra and geometry concepts and skills that are included in the traditional three course series. The problem situations, models, and technology used will foster connections among the various strands of mathematics and develop concepts from multiple perspectives.

Standard 1.0: Number and Operations

Students will recognize, represent, model, and apply real numbers and operations verbally, physically, symbolically, and graphically.

Learning Expectations:

The student will

1.1 demonstrate an understanding of the elements, properties and operations of real numbers;
1.2 demonstrate an understanding of the relative size of rational and irrational numbers;
1.3 connect physical, graphical, verbal, and symbolic representations of real numbers;
1.4 articulate, model and apply the concept of inverse (powers and roots);
1.5 demonstrate an understanding of absolute value;
1.6 recognize the existence of imaginary numbers.
1.7 select and apply an appropriate method (i.e. mental arithmetic, paper and pencil, or technology) for computing with real numbers, and evaluate the reasonableness of results;
1.8 apply matrix operations to solve real-world problems, using appropriate technology.

Student Performance Indicators:

At Level 1, the student is able to

- approximate pi given a table of values for the circumference and diameter of circles;
- order a set of rational and irrational numbers;
- find an integral power of a positive rational number (exponents 1-3).

At Level 2, the student is able to

- use absolute value to express the distance between two points on a number line and vice versa;
- simplify a radical (radicand less than 1000);
- match a given irrational number to the appropriate point on a number line and vice versa (e.g., $\sqrt{2}$, $\sqrt{30}$, pi).

At Level 3, the student is able to

- use radicals and decimal approximations of irrational numbers to indicate calculated lengths or distances;
- represent irrational numbers as lengths of lines in the coordinate plane (e.g. ,$\sqrt{5}$ is the length of the diagonal of a rectangle with base 1 and height 2).

Sample Task: Students will compute the hypotenuse of a given right triangle and arrange themselves in order from smallest to largest.

Linkages: Mathematics: Estimation, Measurement, and Algebra.

Standard 2.0: Algebra

Students will recognize, extend, create, and analyze a variety of geometric, spatial, and numerical patterns; solve real-world problems related to algebra and geometry; and use properties of various geometric figures to analyze and solve problems.

Learning Expectations:

The student will:

2.1 solve systems of three equations and three unknowns using a variety of techniques including inverse matrices with technology;
2.2 describe the domain and range of a function;
2.3 represent real-world problems involving sets, their intersections, union, and complements using Venn diagrams;
2.4 apply Venn diagrams in problem solving;
2.5 solve quadratic equations and inequalities using appropriate methods;
2.6 solve radical equations using appropriate methods;
2.7 graph absolute value functions and quadratic functions with emphasis on transformations;
2.8 solve real-world problems modeled by absolute value or quadratic functions;
2.9 recognize the conic sections from given information;
2.10 recognize, extend, and create numerical, geometric, and spatial patterns;
2.11 generalize patterns verbally and symbolically using function notation.

At Level 1, the student is able to

- extend or find missing element(s) in a geometric patterns and situations (e.g., Fibonacci sequence and Golden Ratio);
- solve multistep linear equations to find length, width, perimeter, and area of geometric figures;
- apply the concept of rate of change to solve a real-world problem given a pattern of data;
- determine the slope given a graph of a linear equation and vice versa;
- determine the distance, midpoint, or slope when given the coordinates of two points (answers must be given as decimals to the nearest hundredth).

At Level 2, the student is able to

- determine the equation of a line parallel or perpendicular to a given line, from given information (e.g., equations of lines, graphs of lines, or two points);
- apply ratio and proportion to solve real-world problems involving polygons, (e.g., scale drawings, similar figures);
- apply the triangle inequality property to determine which sets of side lengths determine a triangle;
- determine the perimeter, area, or volume given the ratio of two similar polygons or rectangular solids;
- apply the Triangle Sum Theorem or Exterior Angle Theorem to determine the measures of the angles of a given triangle with the angle measures expressed algebraically.

At Level 3, the student is able to

- determine the equation of a circle given coordinates or the graph of the circle (e.g., the center, the endpoints of the diameter);
- use manipulatives to determine relationships between linear, square, or cubic measures when one of the measures of the object has changed and represent algebraically.
- apply the line of best fit given real-world data from geometric figures using technology (e.g., finding the interior angle sum of polygons when given the number of sides; find the circumference of circles when given the diameter).
- recognize complete and incomplete networks;
- graph plane figures on a coordinate plane and solve problems algebraically.

Sample Task: Students construct designs using basic geometric constructions. Then they transfer the design to a piece of 8""X 11""pane of plexiglass and paint the pane to create a "stained glass."

Linkages: Mosaic Tiling.

Standard 3.0: Geometry

Students will investigate, model, and apply geometric properties and relationships and use indirect reasoning to make conjectures; deductive reasoning to draw conclusions; and both inductive and deductive reasoning to establish the truth of statements.

Learning Expectations:

The student will:

3.1 demonstrate an understanding of geometric transformations (i.e. reflection, translation, rotation, and dilation);
3.2 apply deductive reasoning using postulates and theorems to prove conclusions from given hypotheses;
3.3 determine the truth of an implication, its converse, inverse, and contrapositive;
3.4 apply right triangle properties, including geometric mean, The Pythagorean Theorem, special right triangles, and the trigonometric ratios;
3.5 derive the distance formula for the distance between two points in a rectangular coordinate system;
3.6 apply concepts related to similar and congruent triangles;
3.7 apply properties of circles, arcs, chords, tangents, or secants to solve problems;
3.8 apply the distance and midpoint formulas in solving problems;
3.9 solve real-world problems involving area with two- and three- dimensional shapes;
3.10 use coordinates to describe position in two and three dimensions.

Student Performance Indicators:

At Level 1, the student is able to

- identify corresponding parts of similar and congruent geometric figures given a diagram;
- determine the length of a missing side in a right triangle when given two sides (answers must be given as simplified radicals).

At Level 2, the student is able to

- identify properties of plane figures from information given in a diagram;

- identify chords, inscribed angles, or central angles of circles given a diagram;
- determine congruence or similarity relations between triangles or quadrilaterals given a diagram;
- determine whether a plane figure has been translated, dilated, reflected, or rotated given a diagram and vice versa;
- solve problems involving complementary, supplementary, congruent, vertical, or adjacent angles given angle measures expressed algebraically;
- determine the trigonometric ratio for a right triangle needed to solve a real-world problem given a diagram;
- find a missing side length in a 30-60-90 or 45-45-90 degree triangle without rationalizing the denominator
- apply properties of quadrilaterals to solve a real-world problem given a diagram (opposite sides and angles, consecutive sides and angles, or diagonals);
- solve real-world problems involving measures of interior or exterior angles of regular polygons;
- identify the appropriate segment of a triangle given a diagram and vice versa (i.e. median, altitude, angle bisector, perpendicular bisector);
- determine which three-dimensional solid is represented by a given net and vice versa (two-dimensional drawing);
- determine the area of indicated regions involving circles, squares, rectangles, and/or triangles;
- justify triangle congruence given a diagram (i.e., ASA, SSS, AAS, SAS, or Hypotenuse/ Leg);
- determine if a triangle is a right triangle given the length of all the sides of a triangle.
- investigate and apply the properties of angles, arcs, chords, tangents, and/or secants using technology or manipulatives; find the area of a sector of a circle given a diagram.
- use inductive and deductive reasoning to make conjectures, draw conclusions, and solve problems;
- recognize and articulate relationships among families of geometric figures (e.g., quadrilaterals, prisms).

At Level 3, the student is able to

- use coordinates to communicate the location of a three-dimensional figure that has been rotated or reflected;
- write and defend indirect and direct proofs;
- use logical reasoning to solve problems in the real world;
- use manipulatives to explore the geometric mean of similar triangles;
- use appropriate tools or technology to develop geometric and spatial concepts;
- construct three-dimensional objects using physical materials and manipulatives;
- compare and construct quadrilateral properties using a variety of models (e.g., Venn diagrams, family trees, manipulative mobiles).

Sample Task: Students construct and use a hypsometer to measure several tall structures on the school grounds.

Linkages: Mathematics: Measurement. Surveying and Art.

Standard 4.0: Measurement

Students will apply appropriate units of measurement; develop effective estimation and computation strategies for solving real world problems involving length, area, and volume; and choose appropriate techniques and tools to measure quantities in order to meet specifications for precision, accuracy, and tolerance.

Learning Expectations:

The student will:

4.1 choose appropriate techniques and tools to measure quantities in order to meet specifications for tolerance;
4.2 perform operations on algebraic expression and informally justify the procedures chosen;
4.3 use concepts of length, area, and volume to estimate and solve real-world problems;
4.4 apply measurement concepts and relationships in algebraic and geometric problem-solving situations;
4.5 use estimation to make predictions and determine reasonableness of results;
4.6 demonstrate an understanding of rates and other derived and indirect measurements (e.g. velocity, miles per hr, revolutions per minute, cost per unit);
4.7 apply geometric properties in constructions using a variety of tools (e.g. paper folding, geometric software, reflections tools).

Student Performance Indicators:

At Level 1, the student is able to

- determine the perimeter or area of a triangle or rectangle when the dimensions are given as first degree binomials in one variable;
- determine the measure of an angle using a protractor.
- solve real world problems involving perimeter or area of three or four sided plane figures.

At Level 2, the student is able to

- determine the volume or surface area of a rectangular solid or cylinder in a real-world situation;
- construct bisectors of angles and line segments, perpendicular lines, congruent line segments and angles, and perpendicular bisectors using a variety of methods (e.g., patty paper, technology).

At Level 3, the student is able to

- determine whether a reading falls within an acceptable tolerance range.
- choose appropriate techniques and tools to measure quantities in order to meet specification for precision, accuracy, and tolerance;
- locate the irrational numbers $\sqrt{2}$ and $\sqrt{3}$ on a number line by using the Pythagorean relationship and a straightedge and compass, manipulatives, or technology;
- solve problems involving surface area of pyramids, cones, and spheres.

Sample Task: Students construct designs using basic geometric constructions. Then they transfer the design to a piece of 8""X 11""pane of plexiglass and paint the pane to create a "stained glass." Students construct one of the regular 3-dimensional solid and compute the volume and surface area.

Linkages: Mathematics – Geometry and Number & Operations. Surveying, construction, and architecture. .Mosaic Tiling.

Standard 5.0: Data Analysis and Probability

Students will investigate, explore, and apply geometric representations to calculate theoretical probability; and will use data from geometric figures to investigate relationships.

Learning Expectations:

The student will:

5.1 demonstrate an understanding of different sampling methods and when each is appropriate;
5.2 use simulations to demonstrate probability experiments;
5.3 use a variety of techniques to determine equations of best fit for quadratic data sets;
5.4 analyze the validity of statistical conclusions;
5.5 determine the probability of an event;
5.6 determine the probability of mutually exclusive events.

Student Performance Indicators:

At Level 1, the student is able to

- make a prediction from a geometric representation of a real-world data set;

At Level 2, the student is able to

- determine the probability of an event represented as a subset of the area of a two-dimensional geometric figure.
- collect and analyze data to make conjectures about geometric relationships.

Sample Task: Construct two 1' X 1' dart boards and draw circular targets on each that are externally tangent to each adjacent circle and to the edge of the board. Draw two circles on one dartboard and three on the other. Throw randomly and count the throws that hit the board to determine which board yields the highest probability of a dart's landing in a circle. Calculate the probability for each bard.

Linkages: Game theory.

Integrated Mathematics III

Course Description:

This course is the second of three courses in a series that uses a more integrated approach to cover the same algebra and geometry concepts and skills that are included in the traditional three course series. The problem situations, models, and technology used will foster connections among the various strands of mathematics and develop concepts from multiple perspectives.

Standard 1.0: Number and Operations

Students will recognize, represent, model, and apply real numbers and operations verbally, physically, symbolically, and graphically.

Learning Expectations:

The student will:

1.1 demonstrate an understanding of the laws of exponents, including integral and rational exponents;
1.2 demonstrate an understanding of the elements, subsets, and properties of the complex number system.
1.3 select and apply an appropriate method (i.e. mental arithmetic, paper and pencil, or technology) for computing with real numbers, and evaluate the reasonableness of results;
1.4 perform operations on algebraic expression and justify the procedures chosen;
1.5 perform operations on complex numbers of the form a + bi.

Student Performance Indicators:

At Level 1, the student is able to

- order a given set of real numbers;
- identify the reciprocal of a real number; probe the relationships among various subsets of the real-number system;
- explore various representations of absolute value on a number line;
- multiply two polynomials with each factor having no more than two terms.

At Level 2, the student is able to

- perform basic operations using complex numbers (i.e., addition, subtraction, and multiplication);
- identify the exponential form of a logarithmic expression and vice versa;
- simplify expressions with rational and negative exponents;
- add, subtract, and multiply algebraic expressions.
- compare and contrast the GCF and the LCM of a set of algebraic expressions;
- add, subtract, and perform scalar multiplication on matrices using appropriate technology.
- use the inverse notation of powers and roots;
- perform basic operations on rational algebraic expressions.

At Level 3, the student is able to

- determine the conjugate of a complex number;
- use delta notation to represent the rate of change in a real-world situation
- justify the procedures chosen when performing operations on algebraic expressions and equations;
- use factorial notation for coefficients in a binomial expansion;
- determine the multiplicative inverse of a complex number;
- formulate the representation of a series using sigma notation.

Sample Task: Students design and build a simple fractal from available materials.

Linkages: Mathematics – Estimation, Measurement, and Computation. Make connections to concept mapping in literature, language arts, and social studies. Connect estimation and computation strategies to business and finance.

Standard 2.0: Algebra

Students will describe, extend, analyze, and create a wide variety of patterns and functions using appropriate materials and representations in real-world problem solving, and will demonstrate an understanding of the behavior of a variety of functions and their graphs.

Learning Expectations:

The student will:

2.1 perform operations on functions, including composition, and determine the effects of the composition on the domain and range;
2.2 demonstrate an understanding of the inverse of a function and determining if the inverse is a function;
2.3 identify and describe the characteristics of families of functions;
2.4 articulate the results of varying parameters of a parent function;
2.5 solve polynomial equations and inequalities using appropriate technology;
2.6 solve absolute value equations and inequalities;
2.7 graph polynomial, exponential, and logarithmic and rational functions;
2.8 solve exponential, logarithmic, and rational equations using appropriate methods and technology;
2.9 solve real-world problems modeled by polynomial, exponential, logarithmic, and periodic functions;
2.10 solve problems involving linear programming;
2.11 demonstrate an understanding of recursive and explicit definitions of functions and sequences;
2.12 recognize the difference between continuous and discrete situations;
2.13 apply sigma notation with arithmetic and geometric series;
2.14 represent a sequence using a list, graph, symbols, and words;
2.15 determine an equation of a conic section from its graph.

Student Performance Indicators:

At Level 1, the student is able to

- translate a verbal sentence into an algebraic equation and vice versa;
- select the algebraic equation that generalizes the pattern represented by data in a given table;
- solve multi-step (more than two steps) linear equations (one set of parentheses on each side of the equations and/or variables on both sides);
- select the graph that represents a given linear function expressed in slope-intercept form;

- select the graph that models a given real-world situation (i.e., linear and non-linear);
- explain what the changes in slope of a non-linear graph represent in a real-world situation;
- analyze mathematical patterns related to algebra and geometry in real-world problem solving.
- identify the graphical representation of the solution to a one-variable inequality on a number line.

Level 2, the student is able to

- select functional notation to generalize a given numeric pattern;
- solve one-variable linear equations with rational expressions;
- select the graph of a two-variable inequality;
- determine the domain of polynomial, rational, square root, exponential and logarithmic functions;
- determine the range of a wide variety of functions given a graph;
- solve a system of linear equations with 2 variables (e.g. substitution, elimination, Cramer's Rule, and graphing);
- apply properties of logarithms to simplify a logarithmic expression;
- identify matrices that model given real-world situations.
- use a variety of methods to solve linear systems in two and three variables (e.g., elimination, substitution, Cramer's Rule, matrices, and graphing);
- explain the restrictions on the variable in a radical equation;
- choose an appropriate method to find the roots of a quadratic equation (e.g. completing the square, quadratic formula, factoring, or graphing calculator);
- solve quadratic inequalities.

At Level 3, the student is able to

- determine the inverse of a logarithmic function given its graph.
- evaluate the graph of a function to determine if it is periodic;
- sketch a system of linear inequalities and determine the maximum or minimum value of the related function;
- justify the procedures chosen when performing operations on algebraic expressions and equations;
- find the maximum or minimum value given the graph of the feasible region of the real world linear programming application;
- determine all the roots of a higher order polynomial (i.e., Descartes' Rule of Signs, Rational Root Theorem, and Synthetic Division).

Sample Task: Examine patterns found in Pascal's Triangle.

Linkages: Mathematics: Statistics and Probability. Data analysis and pattern recognition in science.

Standard 3.0: Geometry

The student will:

Learning Expectations:

3.1 apply and justify properties of quadrilaterals and circles;
3.2 solve real world problems involving volume of geometric solids;
3.3 demonstrate an understanding of the Platonic Solids;
3.4 demonstrate an understanding of uniqueness through indirect proofs;

3.5 apply transformational matrices to transform geometric figures in a rectangular coordinate system.

Student Performance Indicators:

At Level 1, the student is able to

- apply the given Pythagorean Theorem to real-world problems.

At Level 2, the student is able to

- predict the graphical transformation that occurs when coefficients and/or constants of given function are changed (no trigonometric or logarithmic functions);
- apply proportion and the concepts of similar triangles to solve real world problems;
- . estimate the irrational solution of a real-world problem using the Pythagorean Theorem.

At Level 3, the student is able to

- describe the transformation that has changed a "parent function" to the given related function (e.g., right shift of 3 units, reflection in the x-axis ;
- apply the distance formula to obtain the equation of a circle in order to solve real-world problems;
- use deductive reasoning to draw conclusions.
- use matrices to find the area of a triangle on a coordinate plane;
- investigate and explore the conics section.

Sample Task: Students use properties of similar triangles to determine the height of objects that are difficult to measure.

Linkages: Research and discuss geometric applications such as art and use logical reasoning to solve problems in the real world. Use manipulatives to explore the geometric mean of similar triangles; use appropriate tools or technology to develop geometric and spatial concepts; construct three-dimensional objects using physical materials and manipulatives; and compare and construct quadrilateral properties using a variety of models (e.g., Venn diagrams, family trees, manipulative mobiles).

Standard 4.0: Measurement

The student will:

4.1 use concepts of length, area, and volume to estimate and solve real-world problems;
4.2 apply measurement concepts and relationships in algebraic and geometric problem-solving situations;
4.3 use estimation to make predictions and determine reasonableness of results;
4.4 demonstrate an understanding of rates and other derived and indirect measurements (e.g. velocity, miles per hr, rpm, cost per unit).

Student Performance Indicators:

- select the appropriate unit of measure given the real world situation.
- select the area representation for a given product of two binomials.

At Level 2, the student is able to

- apply the given formula to find area and circumference of circles, area and perimeter of polygons, and volume of regular solids;
- use appropriate measurements in collecting data for a real world situation

At Level 3, the student is able to

- solve real world problems given logarithmic and exponential formulas (e.g. Ph scale, Richter scale.).

Sample Task: Construct a regular geometric solid and determine the surface area, volume, and edge length.

Linkages: Science, art, construction, manufacturing.

Standard 5.0: Data Analysis and Probability

Students will collect, organize, represent, and interpret data; make and evaluate inferences and predictions; present and evaluate arguments based on data analysis; and model situations to determine theoretical and experimental probabilities.

Learning Expectations:

The student will:

5.1 describe and apply the normal distribution and its properties;
5.2 use z-scores to compare normally distributed data sets;
5.3 use a variety of techniques to determine equations of best fit for nonlinear data sets;
5.4 calculate and interpret z-scores;
5.5 apply the properties of conditional probability;
5.6 determine binomial probabilities using appropriate methods;
5.7 make inferences about a data set using appropriate measures of central tendency and dispersion, including variance and standard deviation;
5.8 calculate expected value to make judgments about real-life situations.

Student Performance Indicators:

At Level 1, the student is able to

- make a prediction from the graph of a real-world data set;
- determine the measures of central tendency for a given set of real-world data;
- choose the matching linear graph when given a set of ordered pairs representing real-world data.
- analyze student-collected data to make predications or generalizations.

At Level 2, the student is able to

- categorize the correlation of a scatterplot using real-world data (i.e., positive, negative, strong, or weak);
- determine the number of possible outcomes for a given experiment (i.e. the multiplication counting principle, permutations, or combinations);
- determine the theoretical probability of a simple event for a given situation;

- use simulations to help predict the probability of a given situations;
- determine the theoretical probability of a compound event (i.e., dependent or independent, union and intersection);
- determine the theoretical probability of mutually exclusive events for a given situation;
- analyze theoretical or experimental probability to determine the likelihood of an event;
- analyze data using linear and quadratic functions using the appropriate technology;
- analyze the validity of statistical conclusions and the use, misuse, and abuse of data;
- identify the mean and the standard deviation given the graph of a normal distribution.

At Level 3, the student is able to

- find the equation for the line of best fit given a scatterplot depicting real-world data
- use the measure of central tendency which best represents the given real-world data set given a distribution curve.

Sample Task: Students analyze real-world data collected from the newspaper and explore and report the uses, misuses, and abuses of reported statistical data. Students search the internet to collect age and market value of a selected vehicle over a specific period of time. They use a graphing calculator to create a scatterplot and construct a line of best fit to predict the depreciation of the vehicle.

Linkages: Business and Economics; Sports; Social Studies, Science.

Advanced Algebra with Trigonometry

Course Description:

Advanced Algebra with Trigonometry is an advanced mathematics course that extends algebraic concepts and applications and that develops trigonometric functions and applications. Through meaningful problems and appropriate technologies, students will study a variety, radical, rational, polynomial, exponential, circular and logarithmic functions.

Standard 1.0: Algebra

Students will extend algebraic concepts to model and solve problems in real-world situations by using a variety of functions, equations, and inequalities.

Learning Expectations:

The student will:

1.1 represent situations that involve variable quantities with expressions, equations, inequalities, and matrices;
1.2 use appropriate methods and technologies to represent and characterize the solutions for a variety of equations, inequalities, and systems of equations and systems of inequalities;
1.3 demonstrate understanding of sequences and series.

Student Performance Indicators:

- represent (graphically, algebraically, verbally, and numerically) and analyze a variety of functions (polynomial, rational, exponential, and logarithmic) and their characteristics
- graph a variety of functions using transformations;
- solve a variety of equations using appropriate methods;
- solve linear, quadratic, and polynomial inequalities using appropriate methods;
- solve real-world problems modeled by rational, polynomial, exponential, and logarithmic functions;
- use data analysis techniques to model real-world phenomena using functions.
- demonstrate an understanding of operations on matrices;
- solve real-world problems involving networks, finite graphs, and geometric transformations;
- recognize the difference between continuous and discrete situations;
- demonstrate an understanding of recursive and explicit definitions of functions and sequences;
- use sigma notation to represent arithmetic and geometric series;
- represent a sequence using a list, graph, symbols, and words;
- solve problems using permutations and combinations;
- apply the Binomial Theorem to expansion of binomials
- use the discriminants of quadratic equations to characterize the nature of the solutions to the equations and the x-intercepts of the graphs of the equations;
- write equations for conic sections and identify characteristics of the related graphs;
- recognize direct- and inverse-variation situations and solve real-world problems involving variation;
- graphs equations of the forms $y = kx$, $y = kx^2$, $y = k/x$ and $y = k / x^2$.

Standard 2.0: Trigonometry

Students will demonstrate an understanding of trigonometric functions and apply them to problem situations and real-world phenomena.

Learning Expectations:

The student will:

2.1 apply trigonometry concepts and applications to problem situations;
2.2 connect trigonometric and circular functions;
2.3 interpret trigonometric functions represented graphically.

Student Performance Indicators:

- define the trigonometric functions using the unit circle;
- determine values of the trigonometric functions for special angles using the unit circle and the symmetry of the circle;
- graph the trigonometric functions;
- understand amplitude, period, phase shift, and vertical shift and apply to graphing trigonometric functions;
- use trigonometric functions with appropriate technology to model periodic phenomena;
- verify trigonometric identities graphically and by substitution;
- solve trigonometric equations graphically
- use degrees and radians interchangeably to represent angle measure in problems and explain the advantages/disadvantages of a particular choice;
- solve real-world problems applying the trigonometric ratios, the Law of Sines, and the Law of Cosines;
- apply the trigonometric formulas for finding the areas of triangles and circular sectors and segments;
- derive the Pythagorean Identities.

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Discrete Mathematics with Statistics and Probability

Course Description:

Discrete Mathematics with Statistics and Probability is an advanced course that includes using discrete structures to represent and determine solutions to problem situations; collecting, representing, and processing data; and applying probability in problem-solving.

Standard 1.0: Discrete Mathematics

Students will investigate meaningful problems individually or in cooperative groups, while using appropriate technology, to apply discrete structures to represent and solve problems.

Learning Expectations:

The student will:

1.1 use discrete structures to represent problem situations;
1.2 apply inductive and deductive reasoning to discrete problem situations;
1.3 apply discrete ideas and structures to solve a variety of problems.

Student Performance Indicators:

- apply discrete ideas to solve real-world problems (i.e. election theory, group ranking, and estate planning);
- demonstrate an understanding of the elements, subsets, properties, operations of sets;
- use valid forms of deductive reasoning and logic to make and evaluate arguments;
- represent and solve problems using discrete structures such as finite graphs, matrices, and sequences (e.g. Leslie Model, Leontief Model, Markov Chain, and cryptographic techniques);
- use vertex-edge graphs to solve network problems such as finding circuits, critical paths, minimum spanning trees, and adjacency matrices;
- analyze and use discrete ideas such as induction, iteration, and recurrence relations to solve problems from such fields as Chaos Theory, Map Problems, and fractals;
- create tessellations using reflection, rotation, and translation.

Standard 2.0: Probability

Students will expand basic concepts of probability and apply those concepts to represent and solve problems.

Learning Expectations:

The student will:

1.1 demonstrate an understanding of probability distributions;
1.2 apply experimental and theoretical probability in problem solving;

Student Performance Indicators:

- apply number theory topics such as the Fundamental Theorem of Arithmetic, lowest common denominator, greatest common factor, etc. to solve problems using modular arithmetic;
- create and interpret discrete probability distributions;

- use experimental or theoretical probability, as appropriate, to represent and solve problems involving uncertainty;
- derive and use formulas to calculate combinations and permutations;
- understand and apply the concept of a random variable to generate and interpret probability distributions including binomial, uniform, normal, and Chi Square; apply game theory to problem solving.

Standard 3.0: Statistics

Students will select and use appropriate representations and statistical methods to analyze data collected from real-world situations.

Learning Expectations:

The student will:

3.1 formulate questions and design appropriate studies;
3.2 select and use appropriate representations to summarize data;
3.3 select and use appropriate statistics to analyze data.

Student Performance Indicators:

- design a statistical experiment to study a problem, conduct the experiment, and communicate and interpret the outcomes;
- understand and apply measures of central tendency, variability, and correlation to summarize data and draw inferences from real-world situations;
- understand sampling and recognize its role in statistical claims;
- conduct and interpret tests for significance using appropriate statistics;
- use curve fitting to make predictions from data;
- construct and draw inferences from charts, tables, and graphs that summarize data from real-world situations.

Pre-Calculus

Course Description:

Precalculus is an advanced mathematics course that uses meaningful problems and appropriate technologies to build upon previously learned mathematical concepts to develop the underpinnings of calculus.

Standard 1.0: Models for Real-World Phenomena

Students will model and analyze real-world phenomena using techniques from algebra and data analysis.

Learning Expectations:

The student will:

1.1 select and use appropriate algebraic functions to model real-world situations;
1.2 select and use appropriate techniques from data analysis to model real-world phenomena.

Student Performance Indicators:

- model real-world phenomena using techniques of data analysis;
- recognize and apply mathematical models of linear, quadratic, exponential, logarithmic, and trigonometric functions;
- use scatterplot residuals, and/or correlation coefficients to determine whether a model is appropriate;
- apply equations and graphs of conic sections to model real-world phenomena.
- use models when appropriate to draw conclusions or make predictions.

Standard 2.0: Algebraic Functions

Students will extend the concepts of function from earlier courses to a wider variety of functions and their graphs and real-world applications.

Learning Expectations:

The student will:

2.1 represent a variety of functions graphically;
2.2 use a variety of methods to analyze and interpret functions;
2.3 determine the slope and equations of lines tangent to curves;
2.4 apply functions in problem situations.

Student Performance Indicators:

- sketch the graphs of the basic functions (linear, quadratic, cubic, square root, absolute value, reciprocal, trigonometric, exponential, logarithmic, and greatest integer);
- graph transformations and combinations of transformations for all basic functions;
- analyze functions, such as by decomposing into simpler functions;
- determine if a function is even, odd, or neither;
- use an appropriate technology to solve inequalities;

- demonstrate an understanding of the concept of the limit of a function;
- apply the limit of a function to find the slope of a line tangent to a curve;
- write equations of tangents and normals to conic sections;
- apply limits to develop the concept of continuity and identify intervals of increase and decrease;
- locate critical points on the graphs of polynomial functions and determine if each critical point is a minimum, a maximum, or a point of inflection;
- determine an equation of a rational function from a written description.
- define and use the logarithmic function as the inverse of the exponential function;
- sketch the graphs of exponential and logarithmic functions;
- solve exponential and logarithmic equations modeling real-world problems (e.g. growth and decay).

Standard 3.0: Trigonometric Functions

The student will

3.1 apply trigonometry concepts and applications to model and solve problems;
3.2 use trigonometric concepts to represent, apply, and operate with complex numbers;
3.3 solve trigonometric equations and inequalities algebraically or graphically;
3.4 interpret transformations of trigonometric functions.

Student Performance Indicators:

- define six circular functions;
- sketch graphs of the six trigonometric functions involving period change, amplitude change, phase shift, and/or vertical shift;
- use trigonometric functions to model periodic phenomena;
- use graphs to develop and verify trigonometric identities;
- find values of inverse trigonometric functions, applying appropriate domain and range restrictions;
- solve trigonometric equations and inequalities either algebraically or using graphing technology.
- derive the Law of Sines and the Law of Cosines and apply them to solve problems involving triangles and vectors;
- derive and apply the formulas for the area of a triangle and the sector of a circle;
- understand the relationship between measurements in radians and degrees;
- apply radian measures in problems related to linear and angular velocity;
- understand and apply vectors to solve real world problems;
- represent complex numbers in both rectangular and polar form;
- apply the trigonometric form of complex number in calculations;
- prove and apply DeMoivre's Theorem to find roots and powers of complex numbers.

Standard 4.0: Sequences and Series

Students will develop the concept of limit by examining infinite sequences and series.

Learning Expectations:

The student will:

4.1 represent sequences and series;

4.2 determine, when possible, the sums of infinite series.

Student Performance Indicators:

- demonstrate an understanding of sequences by representing them recursively and explicitly;
- use sigma notation to represent a series;
- determine whether a given series converges or diverges;
- find the sum of an infinite series that converge;
- find the sum of an infinite geometric series;
- use the Binomial Theorem to expand binomials.

Statistics

Course Description:

Statistics is an advanced mathematics course that uses meaningful problems and appropriate technologies to use statistical concepts developed in previous courses to develop more advanced means of statistical analyses, interpretations, and predictions.

Standard 1.0 Experimental Design

Students will design and conduct statistical experiments.

Learning Expectations:

The student will:

1.1 design studies that can be addressed with data;
1.2 collect data based on an appropriate sample.

Student Performance Indicators:

- formulate questions that can be addressed with data;
- describe the role of randomization in surveys and experiments;
- select and use a method such as a survey or an experiment to collect data;
- demonstrate understanding of bias in sampling;
- demonstrate an understanding of the Law of Large Numbers;
- demonstrate an understanding of the probability of independent events and conditional probability;
- using appropriate probability models, design a method for simulating data from a particular situation, and use the generated data to analyze the situation;
- design and conduct a statistical experiment to study a problem, and interpret and communicate the outcomes;
- test hypotheses using appropriate statistics.

Standard 2.0: Data Analysis

The student will select and use appropriate statistical methods to analyze data and to develop and evaluate inferences and predictions based on the data.

Learning Expectations:

The student will:

2.1 select and use appropriate displays to represent and summarize the data collected in statistical studies or experiments;
2.2 select and use appropriate statistical methods to analyze data;
2.3 develop and evaluate inferences and predictions based on data.

Student Performance Indicators:

- construct and interpret charts, tables, and graphs that display univariate and bivariate data;
- calculate and apply measures of central tendency and dispersion in order to make inferences about a data set;
- analyze the effects of data transformations on measures of central tendency and variability;
- calculate and apply the correlation between data sets.
- apply the properties of a normal distribution or a Chi-square distribution in appropriate situations in order to make inferences about a data set;
- demonstrate an understanding of the Central Limit Theorem;
- use curve-fitting with appropriate technology to make regression equations in order to represent a data set algebraically and to make inferences;
- demonstrate an understanding of confidence intervals.

Calculus

Course Description:

Calculus is an advanced mathematics course that uses meaningful problems and appropriate technology to develop concepts and applications related to continuity and discontinuity of functions and differentiation, and integration.

Standard 1.0: Functions

Students will expand the concept of functions to include the analysis and interpretation of both continuous and discontinuous functions in problem situations and the development of the concept of limit.

Learning Expectations:

Students will:

1.1 demonstrate an understanding of the concepts and applications related to a variety of continuous functions;
1.2 calculate and estimate limits;
1.3 represent a variety of functions graphically;
1.4 use graphical representations to demonstrate an understanding of asymptotes;
1.5 use a variety of methods to analyze and interpret functions;
1.6 apply functions in problem situations.

Student Performance Indicators:

- analyze the graphs of polynomial, rational, radical, and transcendental functions using appropriate technology;
- predict and explain the observed local and global behavior of a function;
- calculate limits using algebra;
- estimate limits from graphs or tables of data.
- demonstrate an understanding of asymptotes in terms of graphical behavior;
- describe asymptotic behavior in terms of infinite limits and limits at infinity;
- compare relative magnitudes of functions and their rates of change.
- demonstrate an understanding continuity in terms of limits;
- demonstrate a geometric understanding of graphs of continuous functions.

Standard 2.0: Derivatives

Students will extend the concept of slope of a line to develop the concept of derivative.

Learning Expectations:

The student will:

2.1 define, represent and interpret the concept of derivative;
2.2 use the derivative of a function to characterize the function and vice versa;
2.4 connect the relationships among a function and its first and second derivative;
2.5 apply basic rules for differentiation;
2.6 apply derivatives in problem situations.

Student Performance Indicators:

- represent the concept of the derivative geometrically, numerically, and analytically;
- interpret the derivative as an instantaneous rate of change;
- define the derivative as the limit of the difference quotient;
- articulate the relationship between differentiability and continuity.
- articulate corresponding characteristics of graphs of f and f';
- communicate the relationship between the increasing and decreasing behavior f and the sign of f';
- demonstrate an understanding of the Mean Value Theorem and its geometric consequence;
- translate verbal descriptions into equations involving derivatives and vice versa.
- articulate corresponding characteristics of the graphs of f, f', and f'';
- communicate the relationship between the concavity of f and the sign of f'';
- identify points of inflection;
- analyze curves using the notions of monotonicity and concavity;
- optimization, both absolute (global) and relative (local) extrema;
- model rates of change, including related rates problems;
- use implicit differentiation to find the derivative of an inverse function;
- interpret the derivative as a rate of change in varied applied contexts;
- apply basic rules for the derivative of basic functions and their sum, product, and quotient;
- use the chain rule and implicit differentiation.

Standard 3.0: Integrals

Students will develop the concepts of integrals and their applications.

Learning Expectations:

The student will:

3.1 define and apply basic properties of definite integrals;
3.2 evaluate or approximate define integrals;
3.3 apply techniques of antidifferentiation.

Student Performance Indicators:

- communicate the relationship between a Riemann sum and a definite integral;
- apply basic properties of definite integrals;
- evaluate definite integrals using the Fundamental Theorem;
- apply techniques of antidifferentiation;
- find specific antiderivatives using initial conditions, including applications to motion along a line;
- use separable differential equations in modeling;
- use Riemann sums and the Trapezoidal Rule to approximate definite integrals of functions represented algebraically, geometrically, and by tables of values.